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Self-dual R_k lifts of binary self-dual codes

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ABSTRACT

In this paper, we give a method to lift binary self-dual codes to the ring R_k. The lifting method requires solving a system of linear equations over R_k. This technique is applied to $[14, 7, 4]$ binary self-dual code to obtain self-dual codes over R_2. As Gray images of these codes, a substantial number of $[56, 28, 10]$ self-dual codes are generated. By using the extension theorem given by Bouyuklieva and Bouyukliev in [2], ten new extremal binary self-dual codes of length 58 with new enumerators are found which were not previously known to exist.

1. Introduction

In recent years, there has been a growing interest in codes over various kind of finite rings. And parallel to this, self-dual codes over rings have also gained a large popularity among coding theorists. For some of the works done in this direction we refer to [7,8,22].

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There is a theoretical upper bound for the minimum distance d of an $[n, n/2]$ binary self-dual code. In [17], Rains showed that $d \leq 4\lfloor n/24 \rfloor + 6$ if $n \equiv 22 \pmod{24}$ and $d \leq 4\lfloor n/24 \rfloor + 4$, otherwise. Self-dual codes meeting this bound are called extremal. Conway and Sloane listed the possible weight enumerators of extremal self-dual codes of lengths up to 64 and 72 in [6]. Since then, construction of extremal codes with new weight enumerators has generated a great interest. See [3,5,11,18,15] for some of them.

There are two types of weight enumerators for extremal self-dual codes of length 58 as was described in [6]:

$$W_{58,1} = 1 + (165 - 2\beta)y^{10} + (5078 + 2\beta)y^{12} + \cdots \tag{1}$$

where $0 \leq \beta \leq 82$, and

$$W_{58,2} = 1 + (319 - 24\beta - 2\gamma)y^{10} + (3132 + 152\beta + 2\gamma)y^{12} + \cdots \tag{2}$$

where $0 \leq \beta \leq 11$ and $0 \leq \gamma \leq 159 - 2\beta$. New extremal self-dual codes of length 58 were obtained in [4,12,16,19,20,13]. Recently, in [23], the authors mention the known binary self-dual codes of length 58 and they obtain new ones. Together with the ones added from [13] and [23], the existence of such codes is known for $\beta = 55$ in $W_{58,1}$ and

$$\begin{aligned}
&\text{for } \beta = 0 \text{ with } \gamma \in \{2m | m = 0, 1, 5, 6, 8, 9, 10, 11, 13, 68, 71, 79 \text{ or } 15 \leq m \leq 65\},\\
&\beta = 1 \text{ with } \gamma \in \{2m | m = 13, 18, 53, 58, 63 \text{ or } 21 \leq m \leq 57\},\\
&\text{and } \beta = 2 \text{ with } \gamma \in \{2m | m = 0, 16, 18, 19, 20, 21, 22, 46, 49, 50, 55 \text{ or } 24 \leq m \leq 44\}\\
&\text{in } W_{58,2}.
\end{aligned}$$

In this paper, we describe a method to lift binary self-dual codes to self-dual codes over the ring R_k. The technique as will be explained later just uses elementary tools of linear algebra. As an application of this method, the lifts of $[14, 7, 4]$ binary self-dual code to the ring R_2 are investigated. As a result a family of $(14, 2^{28}, 10)$ R_2 self-dual codes are found. Since the Gray map defined for R_2 preserves self-duality, we obtain a substantial number of $[56, 28, 10]$ binary self-dual codes as images of these codes. Then by applying the extension theorem given by Bouyuklieva and Bouyukliev in [2], new extremal binary self-dual codes of length 58 are constructed.

This paper is organized as follows: In Section 2, the basic properties of the ring R_k are given. Also projections and lifts are explained. Section 3 is about the method developed to lift binary self-dual codes to the ring R_k. And in the last section, we give our new results which are obtained as a combination of the lifting and extension methods.