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Homogeneous weights of matrix product codes over finite principal ideal rings

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ABSTRACT

In this paper, the homogeneous weights of matrix product codes over finite principal ideal rings are studied and a lower bound for the minimum homogeneous weights of such matrix product codes is obtained.

1. Introduction

Matrix product codes over finite fields were introduced in [1]. Many well-known constructions can be formulated as matrix product codes, for example, the $(a|a+b)$-construction, the $(a+x|b+x|a+b+x)$-construction, and some quasi-cyclic codes can be rewritten

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as matrix product codes, see [13]. The reference [1] also introduced non-singular by columns matrices and exhibited a lower bound for the minimum Hamming distances of matrix product codes over finite fields constructed by such matrices. More references on matrix product codes appeared later, e.g., in [9–11,15,16].

Codes over finite rings have also been studied from many perspectives since the seminal work [8]. It was also shown later in [18] that a finite Frobenius ring is suitable as an alphabet for linear coding. Further, Ref. [6] showed that, for any finite ring, there is a Frobenius module which is suitable as an alphabet for linear coding. Inspired by the idea of module coding, Ref. [19] proved that the biggest class of finite rings which are suitable as alphabets for linear coding consists of the finite Frobenius rings.

Finite principal ideal rings form an important subclass of finite Frobenius rings. In particular, all the residue rings $\mathbf{Z}_N$ of integers modulo an integer $N > 1$ are principal ideal rings. It is well known that a finite commutative ring is a principal ideal ring if and only if it is a product of finite chain rings. The reference [17] extended the lower bound obtained in [1] for the minimum Hamming distances of matrix product codes with non-singular by columns matrices over finite fields to the minimum homogeneous weights of matrix product codes over finite chain rings.

In this paper, we consider matrix product codes over finite commutative principal ideal rings, and extend the result on the lower bound for the minimum homogeneous weights of matrix product codes over finite chain rings to matrix product codes over finite commutative principal ideal rings.

In the next section, necessary notations and fundamentals are introduced as preliminaries. In Section 3, we state our main theorem, its corollaries and some remarks. Since the proof of the main theorem is long and technical, it is deferred to Section 4.

2. Preliminaries

In this paper, R is always a finite commutative ring.

For the finite commutative ring R and a positive integer ℓ, any non-empty subset C of R^ℓ is called a code over R of length ℓ, or more precisely, an (ℓ, M) code over R, where $M = |C|$ denotes the cardinality of C; the code C over R is said to be linear if C is an R-submodule of R^ℓ. Recall that the usual Hamming weight w_H on R, i.e., $w_H(0) = 0$ and $w_H(r) = 1$ for all non-zero $r \in R$, induces in a standard way the *Hamming weight* on R^ℓ, denoted by w_H again, and the *Hamming distance* d_H on R^ℓ as follows: $w_H(\mathbf{x}) = \sum_{i=1}^{\ell} w_H(x_i)$ for $\mathbf{x} = (x_1, \cdots, x_\ell) \in R^\ell$, and $d_H(\mathbf{x}, \mathbf{x}') = w_H(\mathbf{x} - \mathbf{x}')$ for $\mathbf{x}, \mathbf{x}' \in R^\ell$. We also let $d_H(C) = \min_{\mathbf{c} \neq \mathbf{c}' \in C} d_H(\mathbf{c}, \mathbf{c}')$. This is known as the *minimum Hamming distance* of the code C.

On the other hand, a *homogeneous weight* on the finite commutative ring R is defined to be a non-negative real function w_h from R to the real number field which satisfies the following two conditions:

- $w_h(r) = w_h(r')$ for $r, r' \in R$, provided $Rr = Rr'$,