

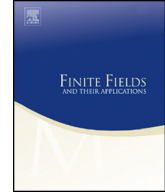


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# On restricted sumsets over a field

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## ARTICLE INFO

### Article history:

Received 19 August 2013

Received in revised form 8 December 2013

Accepted 7 February 2014

Available online 6 March 2014

Communicated by Igor Shparlinski

### MSC:

primary 11B13

secondary 11C08, 11T06

### Keywords:

Sumset

Field

The polynomial method

## ABSTRACT

We consider restricted sumsets over field  $F$ . Let

$$C = \{a_1 + \cdots + a_n : a_1 \in A_1, \dots, a_n \in A_n, a_i - a_j \notin S_{ij} \text{ if } i \neq j\},$$

where  $S_{ij}$  ( $1 \leq i \neq j \leq n$ ) are finite subsets of  $F$  with cardinality  $m$ , and  $A_1, \dots, A_n$  are finite nonempty subsets of  $F$  with  $|A_1| = \cdots = |A_n| = k$ . Let  $p(F)$  be the additive order of the identity of  $F$ . It is proved that  $|C| \geq \min\{p(F), n(k-1) - mn(n-1) + 1\}$  if  $p(F) > mn$ . This conclusion refines the result of Hou and Sun [11].

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## 1. Introduction

Let  $F$  be a field. Denote by  $p(F)$  the additive order of the identity of  $F$ . It is well-known that  $p(F)$  is either infinite or a prime. For a finite set  $A$ , we use  $|A|$  to denote the cardinality of  $A$ .

Suppose that  $A_1, \dots, A_n$  are finite nonempty subsets of  $F$  with  $|A_j| = k_j$  for  $1 \leq j \leq n$ . The Cauchy–Davenport theorem asserts that

$$|\{a_1 + \cdots + a_n : a_1 \in A_1, \dots, a_n \in A_n\}| \geq \min\{p(F), k_1 + \cdots + k_n - n + 1\}.$$

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<http://dx.doi.org/10.1016/j.ffa.2014.02.004>

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Let  $A$  be a finite subset of  $F$ . We define

$$n^{\wedge}A = \{a_1 + \dots + a_n : a_1, \dots, a_n \in A, a_1, \dots, a_n \text{ are distinct}\}.$$

P. Erdős and H. Heilbronn [7] conjectured that

$$|2^{\wedge}A| \geq \min\{p(F), 2|A| - 3\}.$$

This conjecture was solved by Dias da Silva and Hamidoune [5], who established

$$|n^{\wedge}A| \geq \min\{p(F), n|A| - n^2 + 1\}.$$

In 1995–1996, Alon, Nathanson and Ruzsa [2,3] developed the polynomial method to show that if  $0 < k_1 < k_2 < \dots < k_n$ , then

$$|\{a_1 + \dots + a_n : a_i \in A_i, a_1, \dots, a_n \text{ are distinct}\}| \geq \min\left\{p(F), \sum_{j=1}^n (k_j - j) + 1\right\}.$$

Various restricted sumsets of  $A_1, \dots, A_n$  were investigated in [11–16]. In particular, Hou and Sun [11] considered the following sumset

$$C = \{a_1 + \dots + a_n : a_1 \in A_1, \dots, a_n \in A_n, a_i - a_j \notin S_{ij} \text{ if } i \neq j\}, \tag{1.1}$$

where  $S_{ij}$  ( $1 \leq i \neq j \leq n$ ) are finite subsets of  $F$ . Hou and Sun [11] established the following result.

**Theorem 1.1** (*Hou–Sun*). *Let  $C$  be given by (1.1) with  $|S_{ij}| = m$  ( $1 \leq i \neq j \leq n$ ). If  $|A_1| = \dots = |A_n| = k$  and  $p(F) > \min\{n(k - 1) - mn(n - 1), mn\}$ , then*

$$|C| \geq n(k - 1) - mn(n - 1) + 1.$$

The first result of this paper is to refine Theorem 1.1.

**Theorem 1.2.** *Let  $S_{ij}$  ( $1 \leq i \neq j \leq n$ ) be finite subsets of  $F$  with cardinality  $m$ , and let  $C$  be defined in (1.1). Suppose that  $|A_j| \in \{k, k + 1\}$  for  $1 \leq j \leq n$ . If  $p(F) > \min\{mn, \sum_{j=1}^n (|A_j| - 1) - mn(n - 1)\}$ , then*

$$|C| \geq \sum_{j=1}^n (|A_j| - 1) - mn(n - 1) + 1. \tag{1.2}$$

It was pointed out by Hou and Sun [11] that when  $p(F) \leq n(k - 1) - mn(n - 1)$ , Theorem 1.1 only implies

$$|C| \geq n \left\lfloor \frac{p(F) - 1}{n} \right\rfloor + 1, \tag{1.3}$$

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