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Weighted projective lines of tubular type and equivariantization

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We prove that the categories of coherent sheaves over weighted projective lines of tubular type are explicitly related to each other via the equivariantization with respect to certain cyclic group actions.

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1. Introduction

The notion of weighted projective lines is introduced in [7,8], which gives a geometric treatment to the representation theory of the canonical algebras in the sense of [15]. We are interested in weighted projective lines of tubular type. Recall that the category of coherent sheaves over such a weighted projective line is derived equivalent to the module category over a tubular algebra of the same type.

It is known due to [7,11,2] that the category of coherent sheaves over a weighted projective line of tubular type is equivalent to the category of equivariant coherent sheaves over an elliptic curve with respect to a certain cyclic group action; compare [13]. This result explains well that the classification of indecomposable modules over a tubular algebra in [15,12] has similar features as the classification of indecomposable coherent sheaves over an elliptic curve in [1].

In this paper, we show that the categories of coherent sheaves over weighted projective lines of different tubular types are related to each other, via the equivariantization with respect to certain cyclic group actions. Indeed, these cyclic groups are of order two or three, and the actions are the degree-shift actions, which are induced from the grading on the homogeneous coordinate algebras. Here, the equivariantization means forming the category of equivariant objects for a given finite group action on a category; compare [14,4,5].

Let us describe the main results of this paper. Let k be an algebraically closed field, whose characteristic is different from two or three. According to the types, weighted projective lines $\mathbb{X}$ of tubular type are denoted by $\mathbb{X}(2,2,2,2;\lambda)$, $\mathbb{X}(3,3,3)$, $\mathbb{X}(4,4,2)$ and $\mathbb{X}(6,3,2)$, respectively. Here, $\lambda \in k$ is not 0 or 1. The Auslander–Reiten translation on the category coh-$\mathbb{X}$ of coherent sheaves over $\mathbb{X}$ is induced from the degree-shift automorphism by the dualizing element $\vec{\omega}$, which is an element in the grading group of the homogeneous coordinate algebra of $\mathbb{X}$.

In the tubular types, the dualizing element $\vec{\omega}$ has order 2, 3, 4 and 6, according to their types. By the degree-shift automorphisms, we have a strict action on coh-$\mathbb{X}$ by the cyclic group $\mathbb{Z}\vec{\omega}$ and also by its subgroup. For a finite group G and a (strict) G-action on a category $\mathcal{A}$, we denote by $\mathcal{A}^G$ the category of equivariant objects. In particular, we have the category $(\text{coh-}\mathbb{X})^G$ for any subgroup $G \subseteq \mathbb{Z}\vec{\omega}$.

The following theorem combines Propositions 3.2, 3.4 and 3.6. Here, we fix $\epsilon \in k$ satisfying $\epsilon^2 - \epsilon + 1 = 0$.

Theorem. *Keep the notation and assumptions as above. Denote by $\vec{\omega}$ the dualizing element in the grading group of the homogeneous coordinate algebra of $\mathbb{X}$. Then we have the following equivalences of categories.*

(1) $(\text{coh-}\mathbb{X}(4,4,2))^{\mathbb{Z}(2\vec{\omega})} \xrightarrow{\sim} \text{coh-}\mathbb{X}(2,2,2,2;-1)$.
(2) $(\text{coh-}\mathbb{X}(6,3,2))^{\mathbb{Z}(2\vec{\omega})} \xrightarrow{\sim} \text{coh-}\mathbb{X}(2,2,2,2;\epsilon)$.
(3) $(\text{coh-}\mathbb{X}(6,3,2))^{\mathbb{Z}(3\vec{\omega})} \xrightarrow{\sim} \text{coh-}\mathbb{X}(3,3,3)$. $\square$