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On Jones' subgroup of R. Thompson group F

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ABSTRACT

Recently Vaughan Jones showed that the R. Thompson group F encodes in a natural way all knots and links in $\mathbb{R}^3$, and a certain subgroup $\overrightarrow{F}$ of F encodes all oriented knots and links. We answer several questions of Jones about $\overrightarrow{F}$. In particular we prove that the subgroup $\overrightarrow{F}$ is generated by x_0x_1, x_1x_2, x_2x_3 (where x_i, $i \in \mathbb{N}$ are the standard generators of F) and is isomorphic to F_3, the analog of F where all slopes are powers of 3 and break points are 3-adic rationals. We also show that $\overrightarrow{F}$ coincides with its commensurator. Hence the linearization of the permutational representation of F on $F/\overrightarrow{F}$ is irreducible. We show how to replace 3 in the above results by an arbitrary n, and to construct a series of irreducible representations of F defined in a similar way. Finally we analyze Jones' construction and deduce that the Thompson index of a link is linearly bounded in terms of the number of crossings in a link diagram.

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Contents

1. Introduction

A recent result of Vaughan Jones [11] shows that Thompson group F encodes in a natural way all links (this construction is presented in Section 6 below). A subgroup of F, called by Jones the *directed Thompson group* $\overrightarrow{F}$, encodes all oriented links. In order to define $\overrightarrow{F}$, Jones associated with every element g of F a graph $T(g)$ using the description of elements of F as pairs of binary trees (see Section 3 for details). The group $\overrightarrow{F}$ is the set of all elements in F for which the associated graph $T(g)$ is bipartite. Jones asked for an abstract description of the subgroup $\overrightarrow{F}$. For example, it is not clear from the definition whether or not $\overrightarrow{F}$ is finitely generated.

We define the graph $T(g)$ in a different (but equivalent) way. By [5] F is a diagram group. For every diagram Δ in F the graph $T(\Delta)$ is a certain subgraph of Δ. Then, the subgroup of F composed of all reduced diagrams Δ in F with $T(\Delta)$ bipartite is Jones' subgroup $\overrightarrow{F}$. Using this definition we give several descriptions of $\overrightarrow{F}$. Recall that for