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On Jones' subgroup of R. Thompson group  $F$ Gili Golan<sup>1</sup>, Mark Sapir<sup>\*,2</sup>

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## ABSTRACT

Recently Vaughan Jones showed that the R. Thompson group  $F$  encodes in a natural way all knots and links in  $\mathbb{R}^3$ , and a certain subgroup  $\vec{F}$  of  $F$  encodes all oriented knots and links. We answer several questions of Jones about  $\vec{F}$ . In particular we prove that the subgroup  $\vec{F}$  is generated by  $x_0x_1$ ,  $x_1x_2$ ,  $x_2x_3$  (where  $x_i$ ,  $i \in \mathbb{N}$  are the standard generators of  $F$ ) and is isomorphic to  $F_3$ , the analog of  $F$  where all slopes are powers of 3 and break points are 3-adic rationals. We also show that  $\vec{F}$  coincides with its commensurator. Hence the linearization of the permutational representation of  $F$  on  $F/\vec{F}$  is irreducible. We show how to replace 3 in the above results by an arbitrary  $n$ , and to construct a series of irreducible representations of  $F$  defined in a similar way. Finally we analyze Jones' construction and deduce that the Thompson index of a link is linearly bounded in terms of the number of crossings in a link diagram.

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## 1. Introduction

A recent result of Vaughan Jones [11] shows that Thompson group  $F$  encodes in a natural way all links (this construction is presented in Section 6 below). A subgroup of  $F$ , called by Jones the *directed Thompson group*  $\vec{F}$ , encodes all oriented links. In order to define  $\vec{F}$ , Jones associated with every element  $g$  of  $F$  a graph  $T(g)$  using the description of elements of  $F$  as pairs of binary trees (see Section 3 for details). The group  $\vec{F}$  is the set of all elements in  $F$  for which the associated graph  $T(g)$  is bipartite. Jones asked for an abstract description of the subgroup  $\vec{F}$ . For example, it is not clear from the definition whether or not  $\vec{F}$  is finitely generated.

We define the graph  $T(g)$  in a different (but equivalent) way. By [5]  $F$  is a diagram group. For every diagram  $\Delta$  in  $F$  the graph  $T(\Delta)$  is a certain subgraph of  $\Delta$ . Then, the subgroup of  $F$  composed of all reduced diagrams  $\Delta$  in  $F$  with  $T(\Delta)$  bipartite is Jones' subgroup  $\vec{F}$ . Using this definition we give several descriptions of  $\vec{F}$ . Recall that for

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