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Rota's Classification Problem, rewriting systems and Gröbner–Shirshov bases

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A B S T R A C T

In this paper we revisit Rota's Classification Problem on classifying algebraic identities for linear operators. We reformulate Rota's Classification Problem in the contexts of rewriting systems and Gröbner–Shirshov bases, through which Rota's Classification Problem amounts to the classification of operators, given by their defining operator identities, that give convergent rewriting systems or Gröbner–Shirshov bases. Relationship is established between the reformulations in terms of rewriting systems and that of Gröbner–Shirshov bases. We provide an effective condition that gives Gröbner–Shirshov operators and obtain a new class of Gröbner–Shirshov operators.

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1. Introduction

This paper begins general study of Rota's Classification Problem on linear operators.

1.1. Motivation

Motivated by the important roles played by various linear operators in the study of mathematics through their actions on objects, Rota [28] posed the problem of

> finding all possible <u>algebraic identities</u> that can be satisfied by a linear operator on an <u>algebra</u>,

henceforth called **Rota's Classification Problem**.

Operator identities that were interested to Rota included

Endomorphism operator	$d(xy) = d(x)d(y)$,
Differential operator	$d(xy) = d(x)y + xd(y)$,
Average operator	$P(x)P(y) = P(xP(y))$,
Inverse average operator	$P(x)P(y) = P(P(x)y)$,
(Rota–)Baxter operator of weight λ	$P(x)P(y) = P(xP(y) + P(x)y + \lambda xy)$, where λ is a fixed constant,
Reynolds operator	$P(x)P(y) = P(xP(y) + P(x)y - P(x)P(y))$.