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Some generalizations of preprojective algebras and their properties

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ABSTRACT

In this note we consider a notion of relative Frobenius pairs of commutative rings S/R. To such a pair, we associate an $\mathbb{N}$-graded R-algebra $\Pi_R(S)$ which has a simple description and coincides with the preprojective algebra of a quiver with a single central node and several outgoing edges in the split case. If the rank of S over R is 4 and R is Noetherian, we prove that $\Pi_R(S)$ is itself Noetherian and finite over its center and that each $\Pi_R(S)_d$ is finitely generated projective. We also prove that $\Pi_R(S)$ is of finite global dimension if R and S are regular.

Contents

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1. Introduction

1.1. Definitions

For the purposes of this paper, we consider pairs of commutative rings R, S equipped with a map $R \longrightarrow S$. We often write such a pair as S/R. We will always assume R is Noetherian, although some of the results also hold in greater generality.

Definition 1.1. We say that S/R is relative Frobenius of rank n if:

- S is a free R-module of rank n.
- $\mathrm{Hom}_R(S, R)$ is isomorphic to S as S-module.

Remark 1.2.

(i) It is clear that if R is a field, a relative Frobenius pair coincides with a finite dimensional Frobenius algebra in the classical sense.

(ii) Let $e_1, \ldots, e_n$ be any basis for S as an R-module. Then the second condition is equivalent to the existence of a $\lambda \in \mathrm{Hom}_R(S, R)$ such that the R-matrix $(\lambda(e_i e_j))_{i,j}$ is invertible.

(iii) We may equally well assume that S/R is projective of rank n. However all results we prove may be reduced to the free case by suitably localizing R.

We shall need the following notation: for a relative Frobenius pair S/R, let $M := {}_R S_S$. This R–S-bimodule can be viewed as an $R \oplus S$ bimodule by letting the R-component act on the left and the S-component on the right, the other actions being trivial. Similarly, we let $N := {}_S S_R$ and view it as an $R \oplus S$-bimodule by only letting the S-component act on the left and the R-component act on the right, the other actions again begin trivial. We now define

$$T(R, S) := T_{R \oplus S}(M \oplus N)$$

Note that by construction, we have $M \otimes_{R \oplus S} M = N \otimes_{R \oplus S} N = 0$, hence

$$T(R, S)_2 = (M_{R \oplus S} N) \oplus (N \otimes_{R \oplus S} M) = ({}_R S \otimes_S S_R) \oplus ({}_S S \otimes_R S_R)$$