Journal of Algebra 472 (2017) 15–66

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Ladders and simplicity of derived module categories

Lidia Angeleri Hügel [a,1], Steffen Koenig [b], Qunhua Liu [b,c,*,2], Dong Yang [b,d,3]

[a] *Dipartimento di Informatica – Settore Matematica, Università degli Studi di Verona, Strada Le Grazie 15 – Ca' Vignal 2, I-37134 Verona, Italy*
[b] *Institute of Algebra and Number Theory, University of Stuttgart, Pfaffenwaldring 57, 70569 Stuttgart, Germany*
[c] *Institute of Mathematics, School of Mathematical Sciences, Nanjing Normal University, Nanjing 210023, PR China*
[d] *Department of Mathematics, Nanjing University, Nanjing 210093, PR China*

ARTICLE INFO

Article history:
Received 23 August 2016
Available online 24 October 2016
Communicated by Luchezar L. Avramov

Keywords:
Derived module categories
Recollements
Ladders
Lifting and restricting recollements
Derived simple algebras
Homological dimensions
Commutative noetherian rings

ABSTRACT

Recollements of derived module categories are investigated, using a new technique, ladders of recollements, which are maximal mutation sequences. The position in the ladder is shown to control whether a recollement restricts from unbounded to another level of derived category. Ladders also turn out to control derived simplicity on all levels. An algebra is derived simple if its derived category cannot be deconstructed, that is, if it is not the middle term of a non-trivial recollement whose outer terms are again derived categories of algebras. Derived simplicity on each level is characterised in terms of heights of ladders.

These results are complemented by providing new classes of examples of derived simple rings, in particular indecomposable commutative rings, as well as by a finite dimensional counterexample to the Jordan–Hölder problem for derived

* Corresponding author.
E-mail addresses: lidia.angeleri@univr.it (L. Angeleri Hügel), skoenig@mathematik.uni-stuttgart.de (S. Koenig), 05402@njnu.edu.cn (Q. Liu), yangdong@nju.edu.cn (D. Yang).

[1] LAH acknowledges partial support by the University of Padova through Project CPDA105885/10 "Differential graded categories", by DGI MICIIN MTM2011-28992-C02-01, and by the Comissionat Per Universitats i Recerca de la Generalitat de Catalunya through Project 2009 SGR 1389.
[2] QL acknowledges partial support by NSF of China 11301272 and NSF of Jiangsu Province of China BK20130899.
[3] DY acknowledges support by the DFG priority programme SPP 1388 YA297/1-1 and KO1281/9-1.

http://dx.doi.org/10.1016/j.jalgebra.2016.10.023

module categories. Moreover, recollements are used to compute homological and K-theoretic invariants.

Contents

1. Introduction

Derived categories, introduced by Grothendieck and Verdier, have been playing an increasingly important role in various areas of mathematics, including representation theory, algebraic geometry, microlocal analysis and mathematical physics. Major topics