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Ladders and simplicity of derived module categories



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ABSTRACT

Recollements of derived module categories are investigated, using a new technique, ladders of recollements, which are maximal mutation sequences. The position in the ladder is shown to control whether a recollement restricts from unbounded to another level of derived category. Ladders also turn out to control derived simplicity on all levels. An algebra is derived simple if its derived category cannot be deconstructed, that is, if it is not the middle term of a non-trivial recollement whose outer terms are again derived categories of algebras. Derived simplicity on each level is characterised in terms of heights of ladders.

These results are complemented by providing new classes of examples of derived simple rings, in particular indecomposable commutative rings, as well as by a finite dimensional counterexample to the Jordan–Hölder problem for derived

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module categories. Moreover, recollements are used to compute homological and K-theoretic invariants.

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1. Introduction

Derived categories, introduced by Grothendieck and Verdier, have been playing an increasingly important role in various areas of mathematics, including representation theory, algebraic geometry, microlocal analysis and mathematical physics. Major topics

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