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Induced and coinduced modules over cluster-tilted algebras

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ABSTRACT

We propose a new approach to study the relation between the module categories of a tilted algebra C and the corresponding cluster-tilted algebra $B = C \ltimes E$. This new approach consists of using the induction functor $- \otimes_C B$ as well as the coinduction functor $D(B \otimes_C D-)$. We show that DE is a partial tilting and a τ-rigid C-module and that the induced module $DE \otimes_C B$ is a partial tilting and a τ-rigid B-module. Furthermore, if $C = \operatorname{End}_A T$ for a tilting module T over a hereditary algebra A, we compare the induction and coinduction functors to the Buan–Marsh–Reiten functor $\operatorname{Hom}_{\mathcal{C}_A}(T, -)$ from the cluster-category of A to the module category of B. We also study the question as to which B-modules are actually induced or coinduced from a module over a tilted algebra.

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1. Introduction

Cluster-tilted algebras are finite dimensional associative algebras which were introduced in [18] and, independently, in [22] for the type $\mathbb{A}$.

One motivation for introducing these algebras came from Fomin and Zelevinsky's cluster algebras [27]. To every cluster in an acyclic cluster algebra one can associate a cluster-tilted algebra, and the indecomposable rigid modules over the cluster-tilted algebra correspond bijectively to the cluster variables outside the chosen cluster. Generalizations of cluster-tilted algebras, the Jacobian algebras of quivers with potentials, were introduced in [26], extending this correspondence to the non-acyclic types. Many authors have studied cluster-tilted algebras in this context, see for example [14,18–21, 23,24,29].

The second motivation came from classical tilting theory. Tilted algebras are the endomorphism algebras of tilting modules over hereditary algebras, whereas cluster-tilted algebras are the endomorphism algebras of cluster-tilting objects over cluster categories of hereditary algebras. This similarity in the two definitions leads to the following precise relation between tilted and cluster-tilted algebras, which was established in [3].

There is a surjective map

$$\{\text{tilted algebras}\} \longrightarrow\!\!\!\!\!\to \{\text{cluster-tilted algebras}\}, \qquad C \longmapsto B = C \ltimes E,$$

where E denotes the C–C-bimodule $E = \operatorname{Ext}^2_C(DC, C)$ and $C \ltimes E$ is the trivial extension.

This result allows one to define cluster-tilted algebras without using the cluster category. It is natural to ask how the module categories of C and B are related, and several results in this direction have been obtained, see for example [4–6,13,15,25].

The Hochschild cohomology of the algebras C and B has been compared in [7,9,10,31].

In this paper, we use a new approach to study the relation between the module categories of a tilted algebra C and its cluster-tilted algebra $B = C \ltimes E$, namely *induction* and *coinduction*.

The induction functor $- \otimes_C B$ and the coinduction functor $\operatorname{Hom}_C(B, -)$ from $\operatorname{mod} C$ to $\operatorname{mod} B$ are defined whenever C is a subring of B which has the same identity. If we are dealing with algebras over a field k, we can, and usually do, write the coinduction functor as $D(B \otimes_C D-)$, where $D = \operatorname{Hom}(-, k)$ is the standard duality.

Induction and coinduction are important tools in classical Representation Theory of Finite Groups. In this case, B would be the group algebra of a finite group G and C the group algebra of a subgroup of G (over a field whose characteristic is not dividing the group orders). In this situation, the algebras are semi-simple, induction and coinduction are the same functor, and this functor is exact.

For arbitrary rings, and even for finite dimensional algebras, the situation is not that simple. In general, induction and coinduction are not the same functor and, since the C-module B is not projective (and not flat), induction and coinduction are not exact functors.