



Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra



An algebraic approach to the KZ-functor for rational Cherednik algebras associated with cyclic groups



Sam Thelin¹

*Institute of Algebra and Number Theory, University of Stuttgart,
Pfaffenwaldring 57, 70569 Stuttgart, Germany*

ARTICLE INFO

Article history:

Received 27 May 2016

Available online 16 September 2016

Communicated by J.T. Stafford

MSC:

16G99

Keywords:

Rational Cherednik algebras

KZ-functor

Coinvariant algebra

ABSTRACT

We give a complete algebraic description of the KZ-functor for rational Cherednik algebras associated with cyclic groups for a subset of parameter values from which all parameter values can be obtained by integral translations. This is done by identifying the precise parameter values for which the projective object P_{KZ} is isomorphic to the Δ -module associated with the coinvariant algebra and by determining the action of the cyclotomic Hecke algebra on P_{KZ} in this case.

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1. Introduction

The rational Cherednik algebra $H_c(W, \mathfrak{h})$ associated with the complex reflection group W and one of its reflection representations \mathfrak{h} was introduced in [10] using ideas from [6], and is a certain deformation of $S(\mathfrak{h} \oplus \mathfrak{h}^*) \rtimes W$, depending on a finite set

E-mail address: sam.thelin@mathematik.uni-stuttgart.de.

¹ Sections 3–5 of this preprint were part of the authors D. Phil thesis [20] at the University of Oxford, completed in October 2014. I would like to thank my supervisors Karin Erdmann and Raphaël Rouquier for their invaluable support. I also wish to thank Iain Gordon and Ivan Losev for helpful comments. I am grateful to EPSRC for their financial support through grant EP/P505666/1.

of complex parameters c . The algebra $H_c(W, \mathfrak{h})$ has a triangular decomposition (Theorem 1.3 of [10]), reminiscent of that of the universal enveloping algebra of a finite-dimensional semisimple complex Lie algebra \mathfrak{g} . As a result, it has a category $\mathcal{O}_c(W, \mathfrak{h})$ of representations associated with it as introduced in [9] and [2], inspired by and sharing many similarities with the BGG category \mathcal{O} associated with \mathfrak{g} defined in [3]. In particular, it was proved in [13] that category $\mathcal{O}_c(W, \mathfrak{h})$ is a highest weight category in the sense of [8], indexed by the irreducible W -modules. An important problem in the representation theory of rational Cherednik algebras is to understand the structure of category $\mathcal{O}_c(W, \mathfrak{h})$.

A crucial tool to this end is the Knizhnik–Zamolodchikov functor, or KZ-functor for short, introduced in [11]. This functor relates the category $\mathcal{O}_c(W, \mathfrak{h})$ to the category of finitely-generated modules over a corresponding cyclotomic Hecke algebra $\mathcal{H}_c(W, \mathfrak{h})$ (cf. Section 4.C of [5]), and enables results in the representation theory of Hecke algebras to be translated to the setting of rational Cherednik algebras and vice versa. The construction of the functor is geometric in nature, but the functor is exact, and is therefore represented by a projective object P_{KZ} of $\mathcal{O}_c(W, \mathfrak{h})$ (cf. Section 5.4 of [11]). The main aim of this article is to give an explicit algebraic description of P_{KZ} , including the action of the Hecke algebra $\mathcal{H}_c(\mathbb{Z}/n, \mathbb{C})$ on it, in the case of the rational Cherednik algebras $H_c(\mathbb{Z}/n, \mathbb{C})$ corresponding to cyclic reflection groups, for a certain subset of parameter values.

More specifically, we show that in category $\mathcal{O}_c(\mathbb{Z}/n, \mathbb{C})$, there is a subset \mathcal{F} of parameter values for which P_{KZ} is isomorphic to the module $\Delta_c(S(\mathfrak{h})_W)$ corresponding to the coinvariant algebra $S(\mathfrak{h})_W$, and that all possible parameter values can be obtained from \mathcal{F} by integral translations. This has been conjectured by Rouquier to hold for rational Cherednik algebras associated with general complex reflection groups, and was proved in [14] for the groups $G(l, 1, n) = \mathfrak{S}_n \times (\mathbb{Z}/l)^n$. Here we obtain the result independently for cyclic reflection groups. In addition, we establish the precise parameter values for which $P_{\text{KZ}} \cong \Delta_c(S(\mathfrak{h})_W)$, and thus, using the terminology introduced in [14], determine all the totally aspherical parameters for $H_c(\mathbb{Z}/n, \mathbb{C})$. This work was part of the authors D. Phil thesis [20], completed in October 2014. Furthermore, we complete the algebraic description of the KZ-functor in this case, by giving an explicit description of the action of the cyclotomic Hecke algebra $\mathcal{H}_c(\mathbb{Z}/n, \mathbb{C})$ on $\Delta_c(S(\mathfrak{h})_W)$ for parameter values in \mathcal{F} .

The structure of this article is as follows: In Section 2, we include some background material on rational Cherednik algebras, category \mathcal{O} , the KZ-functor and the coinvariant algebra, focusing in particular on the facts relevant to the rational Cherednik algebras $H_c(\mathbb{Z}/n, \mathbb{C})$. We then study maps between the modules $\Delta_c(S(\mathfrak{h})_W)$ and $\nabla_c(S(\mathfrak{h}^*)_W^{\otimes}) = \nabla_c(S(\mathfrak{h}^*)_W \otimes_{\mathbb{C}} \det_{\mathfrak{h}}^{-1}(N))$ for general complex reflection groups W in Section 3 and prove:

Theorem A. *There is a one-to-one correspondence between the elements of the spaces $\mathcal{S} = \text{Hom}_{H_c(W, \mathfrak{h})}(\Delta_c(S(\mathfrak{h})_W), \nabla_c(S(\mathfrak{h}^*)_W^{\otimes}))$ and $\text{Hom}_{\mathbb{C}W}(S(\mathfrak{h}^*)_W, \text{Har}^*)$.*

From Section 4 onwards, we specialise to the rational Cherednik algebras $H_c(\mathbb{Z}/n, \mathbb{C})$ associated with cyclic reflection groups. In this setting, we give a criterion for when an

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