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On Lagrangian algebras in group-theoretical braided fusion categories

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ABSTRACT

We describe Lagrangian algebras in twisted Drinfeld centres for finite groups.
Using the full centre construction, we establish a 1-1 correspondence between Lagrangian algebras and module categories over pointed fusion categories.

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1. Introduction

In this paper, we classify Lagrangian algebras in group-theoretical modular categories. This, in particular, gives a classification of physical modular invariants for group-theoretical modular data (a problem raised in [2]). It should be mentioned that the set of labels for physical modular invariants was described in [12] (using the language of module categories). What was missing was a way of recovering the modular invariant corresponding to a label. By establishing a correspondence between Lagrangian algebras and module categories, and by computing the characters of Lagrangian algebras, we give a method for explicitly computing modular invariants.

By a group-theoretical modular category $\mathcal{Z}(G, \alpha)$, we mean the monoidal (or Drinfeld) centre $\mathcal{Z}(\mathcal{V}(G, \alpha))$ of the category of vector spaces $\mathcal{V}(G, \alpha)$ graded by a finite group G.[1] Here, α is a 3-cocycle of G with coefficients in the multiplicative group k^* of the ground field, which is used to twist the standard associativity constraint for the tensor product of graded vector spaces. More precisely, we define $\mathcal{Z}(G, \alpha)$ to be the category of G-graded vector spaces with compatible G-action (section 3.1), and prove later (section 4.1) that $\mathcal{Z}(G, \alpha)$ is isomorphic to the monoidal centre $\mathcal{Z}(\mathcal{V}(G, \alpha))$.

A commutative algebra A in a braided fusion category $\mathcal{C}$ is Lagrangian if any local A-module in $\mathcal{C}$ is a direct sum of copies of A (we recall basic concepts of braided tensor categories in section 2). We classify Lagrangian and more general indecomposable commutative separable (etale for short) algebras in $\mathcal{Z}(G, \alpha)$ in two steps. First, we describe etale algebras with the trivial grading (section 3.3). These are nothing but indecomposable commutative separable algebras with a G-action and hence are just function algebras on transitive G-sets (we work over an algebraically closed field k). Up to isomorphism, they are labelled by (conjugacy classes of) subgroups $H \subset G$. Then we identify the category of local modules $\mathcal{Z}(G, \alpha)^{\mathrm{loc}}_{k(G/H)}$ with the group-theoretical modular category $\mathcal{Z}(H, \alpha|_H)$ (Theorem 3.6). A general etale algebra in $\mathcal{Z}(G, \alpha)$ is an extension of its trivial degree component, and hence is an etale algebra in one of $\mathcal{Z}(G, \alpha)^{\mathrm{loc}}_{k(G/H)}$. Considered as an algebra in $\mathcal{Z}(H, \alpha|_H)$, it has the one-dimensional trivial degree component. Our second step (section 3.4) is to classify such algebras (Proposition 3.11) and their categories of local modules (Theorem 3.13). Then in section 3.5, we combine the results obtaining the description of all etale algebras in $\mathcal{Z}(G, \alpha)$ (Theorem 3.15) and their local modules (Theorem 3.16). As a corollary, we get a classification of Lagrangian algebras

[1] These are the only group-theoretical modular categories which have Lagrangian algebras.