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Homological properties of determinantal arrangements

Arnold Yim [1]

Department of Mathematics, Purdue University, 150 North University Street, West Lafayette, IN 47907-2067, United States

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We explore a natural extension of braid arrangements in the context of determinantal arrangements. We show that these determinantal arrangements are free divisors. Additionally, we prove that free determinantal arrangements defined by the minors of $2 \times n$ matrices satisfy nice combinatorial properties. We also study the topology of the complements of these determinantal arrangements, and prove that their higher homotopy groups are isomorphic to those of S^3. Furthermore, we find that the complements of arrangements satisfying those same combinatorial properties above have Poincaré polynomials that factor nicely.

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E-mail address: ayim@purdue.edu.

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1. Introduction

In this paper, we investigate a family of hypersurfaces known as *determinantal arrangements*. Determinantal arrangements are unions of hypersurfaces defined by the minors of a matrix of indeterminates. We focus on determinantal arrangements defined by the 2-minors of a $2 \times n$ generic matrix. Since these determinantal arrangements can be thought of as natural generalizations of braid arrangements and graphic arrangements, we aim to extend the results known for these hyperplane arrangements to our new setting. In particular, we study the freeness of these arrangements, and the topology of their complements.

Let D be a divisor in an n-dimensional complex analytic manifold X. The *module of logarithmic derivations* $\mathrm{Der}_X(-\log D) := \{\theta \in \mathrm{Der}_X | \theta(\mathcal{O}_X(-D)) \subseteq \mathcal{O}_X(-D)\}$ are the vector fields on X that are tangent to the smooth points of D. If $\mathrm{Der}_X(-\log D)$ is locally free, then D is called a *free divisor*.

Free divisors were first introduced by Saito [12], motivated by his study of the discriminants of versal deformations of isolated hypersurface singularities. The study of free divisors coming from discriminants of versal deformations has since been a driving force in the theory of singularities (see [9,2,19,18,17]).

Aside from versal deformations, free divisors show up naturally in many different settings. For example, many of the classically arising hyperplane arrangements are free (see [10]). This includes braid arrangements and all Coxeter arrangements.

In general, it is not clear which divisors are free and which are not. Naturally, one might be interested in freeness for arrangements of more general hypersurfaces. For example, Schenck and Tohăneanu [14] give conditions for when an arrangement of lines and conics on $\mathbb{P}^2$ is free.

For determinantal arrangements, Buchweitz and Mond [3] showed that the arrangement defined by the product of the maximal minors of a $n \times (n+1)$ matrix of indeterminates is free. More recently, Damon and Pike [5] showed that certain determinantal arrangements coming from symmetric, skew-symmetric and square generic matrices are free and have complements that are $K(\pi, 1)$. In both of these cases, the arrangements turn out to be linear free divisors (i.e. the basis for $\mathrm{Der}_X(-\log D)$ is generated by linear vector fields). The vector fields arising in these situations correspond to matrix group actions on the generic matrix which stabilize the divisor D. Many interesting determinantal arrangements, however, are not linear free divisors as our next example shows.