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On the Morita Frobenius numbers of blocks of finite reductive groups

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ABSTRACT

We show that the Morita Frobenius number of the blocks of the alternating groups, the finite groups of Lie type in defining characteristic, and the Ree and Suzuki groups is 1. We also show that the Morita Frobenius number of almost all of the unipotent blocks of the finite groups of Lie type in non-defining characteristic is 1, and that in the remaining cases it is at most 2.

1. Introduction

Let ℓ be a prime number, let $k = \overline{\mathbb{F}}_\ell$ be an algebraic closure of the field of ℓ elements and let A be a finite dimensional k-algebra. For $a \in \mathbb{N}$, the *a-th Frobenius twist of A*, denoted by $A^{(\ell^a)}$, is a k-algebra with the same underlying ring structure as A, endowed with a new action of the scalars of k given by $\lambda.x = \lambda^{\frac{1}{\ell^a}} x$ for all $\lambda \in k$, $x \in A$. Two finite dimensional algebras A and B are *Morita equivalent* if $\mathrm{mod}(A)$ and $\mathrm{mod}(B)$ are equivalent k-linear categories. By definition, A and $A^{(\ell^a)}$ are isomorphic as rings, however, they need not even be Morita equivalent as k-algebras. The *Morita Frobenius number* of a k-algebra A, denoted by $mf(A)$, is the least integer a such that A is Morita equivalent to $A^{(\ell^a)}$.

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The concept of Morita Frobenius numbers was introduced by Kessar in [21] in the context of Donovan's Conjecture in block theory. Donovan's Conjecture implies that Morita Frobenius numbers of ℓ-blocks of finite groups are bounded by a function which depends only on the size of the defect groups of the block. Little is known about the values of Morita Frobenius numbers in general, but it is known that a block of a group algebra can have Morita Frobenius number greater than 1 [2]. In this paper we calculate the Morita Frobenius numbers of a large class of blocks of finite reductive groups. We have used GAP [14] to check that the Morita Frobenius number of blocks of simple sporadic groups and their covers is 1. See Sections 2 to 5 for an explanation of the notation in the following theorem.

Theorem 1.1. *Let b be an ℓ-block of a quasi-simple finite group G. Let $\overline{G} = G/Z(G)$. Suppose that one of the following holds*

(a) *$\overline{G}$ is an alternating group;*
(b) *$\overline{G}$ is a finite group of Lie type in characteristic ℓ;*
(c) *$\overline{G}$ is a finite group of Lie type in characteristic not equal to ℓ, b dominates a unipotent block of $\overline{G}$, and b is not one of the following blocks of E_8;*
   - *$b = b_{E_8}(\phi_1^2.E_6(q), E_6[\theta^i])$ $(i = 1, 2)$ with $\ell = 2$ and q of order 1 modulo 4;*
   - *$b = b_{E_8}(\phi_2^2.{}^2E_6(q), {}^2E_6[\theta^i])$ $(i = 1, 2)$ with $\ell \equiv 2 \bmod 3$ and q of order 2 modulo ℓ.*

Then $mf(b) = 1$. In the excluded cases of part (c), $mf(b) \leq 2$.

We start with some general results on the Morita Frobenius numbers of blocks in Section 2. Section 3 deals with the case of the alternating groups, and Section 4 deals with blocks of finite groups of Lie type in defining characteristic. In Section 5 we first present key results from e-Harish Chandra theory and unipotent block theory, followed by the results for unipotent blocks of finite groups of Lie type in non-defining characteristic. In Section 6 the results for exceptional covering groups are presented, and finally the proof of Theorem 1.1 is given in Section 7.

2. General results on Morita Frobenius numbers of blocks

Throughout, ℓ is a prime number, k is an algebraically closed field of characteristic ℓ, and G is a finite group.

2.1. Results on k-algebras

Let A and B be finite dimensional k-algebras and let A_0 and B_0 be basic algebras of A and B respectively. We define the *Frobenius number* of A to be the least integer a such that $A \cong A^{(\ell^a)}$ as k-algebras, and denote it by *frob*(A). Recall that A and B are Morita equivalent if and only if $A_0 \cong B_0$ as k-algebras, and note that $A_0^{(\ell)}$ is a basic