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Primitive permutation groups as products of point stabilizers

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We prove that there exists a universal constant c such that any finite primitive permutation group of degree n with a non-trivial point stabilizer is a product of no more than $c \log n$ point stabilizers.

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1. Introduction

Given a finite group G[4] and a subgroup H of G whose normal closure is G, one can show, by a straightforward elementary argument, that G is the setwise product of at least $\frac{\log|G|}{\log|H|}$ conjugates of H. A far reaching conjecture of Liebeck, Nikolov and Shalev states [8] that in the case that G is a non-abelian simple group, $\frac{\log|G|}{\log|H|}$ is in fact the right order of magnitude for the minimal number of conjugates of H whose product is G, namely, there exists a universal constant c such that for any non-abelian simple group G and any non-trivial $H \leq G$, the group G is the product of no more than $c\frac{\log|G|}{\log|H|}$ conjugates of H. Later on, in [9], this conjecture was extended to allow H to be any subset of G of size at least 2. Some weaker versions of these conjectures are proved in [8, Theorem 2], [9, Theorem 3], and [5, Theorem 1.3].

Here we look for a universal upper bound on the minimal length of a product covering of a finite primitive permutation group by conjugates of a point stabilizer. We will prove the following logarithmic[5] bound:

Theorem 1. *There exists a universal constant c such that if G is any primitive permutation group of degree n with a non-trivial point stabilizer H then G is a product of at most $c \log n$ conjugates of H.*

We note, that in most relevant cases, $\frac{\log|G|}{\log|H|} < \log|G : H| = \log n$ (see Lemma 2.1), so it may well be that the bound provided by Theorem 1 is not the best possible. However, at present it seems hard to resolve this even for one particular O'Nan–Scott family of primitive groups. We believe that these questions deserve further investigation.

2. Preliminaries

We collect some preparatory results and notation.

Lemma 2.1. *Let G be a group and $H \leq G$ such that $|H| \geq 4$ and $|G : H| \geq 4$. Then $\log|G| / \log|H| \leq \log|G : H|$.*

Proof. Set $x := \log|G|$ and $y := \log|H|$. Then the desired inequality reads $x/y \leq x - y$, which is equivalent to $x \geq y + 1 + \frac{1}{y-1}$. Since $y \geq 2$ because $|H| \geq 4$, this is clearly satisfied if $x \geq y + 2$, which is equivalent to $|G : H| \geq 4$. □

Lemma 2.2. *Let G be an almost simple group with socle T. Let M be a maximal subgroup of G and let $M_0 := T \cap M$. Then $|M_0| \geq 6$.*

[4] All groups discussed are assumed to be finite.
[5] Throughout the paper, log stands for logarithm in base 2.