Journal of Algebra 469 (2017) 302–322

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Star-polynomial identities: Computing the exponential growth of the codimensions [☆]

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ARTICLE INFO

Article history:
Received 1 April 2016
Available online 30 August 2016
Communicated by Volodymyr Mazorchuk

MSC:
primary 16R10, 16R50
secondary 16P90

Keywords:
Polynomial identity
Involution
Superinvolution
Codimensions

ABSTRACT

Can one compute the exponential rate of growth of the $*$-codimensions of a PI-algebra with involution $*$ over a field of characteristic zero? It was shown in [2] that any such algebra A has the same $*$-identities as the Grassmann envelope of a finite dimensional superalgebra with superinvolution B. Here, by exploiting this result we are able to provide an exact estimate of the exponential rate of growth $exp^*(A)$ of any PI-algebra A with involution. It turns out that $exp^*(A)$ is an integer and, in case the base field is algebraically closed, it coincides with the dimension of an admissible subalgebra of maximal dimension of B.

☆ The first author was partially supported by CNPq proc. 400439/2014-0, the second author was partially supported by FAPESP, proc. 2005/60411-8 and CNPq proc. 300243/79-0, the third author was partially supported by GNSAGA of INdAM.

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http://dx.doi.org/10.1016/j.jalgebra.2016.07.037
0021-8693/

1. Introduction

Let A be an algebra over a field F of characteristic zero and suppose that A is a PI-algebra i.e., it satisfies a non-trivial polynomial identity. A celebrated result of Kemer states that any such algebra A has the same polynomial identities as the Grassmann envelope of a suitable finite dimensional superalgebra [19]. Recall that if G is the (infinite dimensional) Grassmann algebra over F, one can consider its standard $\mathbb{Z}_2$-grading $G = G_0 \oplus G_1$. Then if $B = B_0 \oplus B_1$ is a superalgebra over F, the Grassmann envelope of B is the algebra $G(B) = G_0 \otimes B_0 \oplus G_1 \otimes B_1$. This result has been extended to algebras graded by a finite group H in [4] (see also [21]). In this case one considers the Grassmann envelope of a finite dimensional $\mathbb{Z}_2 \times H$-graded algebra.

In this paper we are concerned with algebras with involution. In [2] a suitable superinvolution on the Grassmann algebra was introduced having the following property: if B is any algebra endowed with a superinvolution, then its Grassmann envelope has an induced involution. More generally given a superalgebra B and its Grassmann envelope $G(B)$ there is a well-understood duality between graded involutions and superinvolutions of the two algebras. The main outcome of this correspondence is the following result proved in [2]: any PI-algebra with involution $*$ has the same $*$-identities as the Grassmann envelope of a suitable finite dimensional algebra with superinvolution.

Here we are interested in the growth of the identities of an algebra. Recall that if P_n is the space of multilinear polynomials in n variables and $Id(A)$ is the T-ideal of identities of the algebra A, then

$$c_n(A) = \dim P_n/(P_n \cap Id(A))$$

is the n-th codimension of A. It is well known [20] that the sequence of codimensions of an associative PI-algebra is exponentially bounded and in [12,13] it was shown that if A is any PI-algebra there exist constants $C_1 > 0, C_2, t_1, t_2$ such that $C_1 n^{t_1} d^n \leq c_n(A) \leq C_2 n^{t_2} d^n$ holds for a suitable integer d. In particular the limit $\lim_{n\to\infty} \sqrt[n]{c_n(A)} = d = exp(A)$ exists and is an integer called the PI-exponent of A. We refer the reader to [15] for an account of the theory developed around the exponent.

We have to mention that an actual asymptotic estimate of the codimensions was established in [7] and [8] for algebras with 1 and it turns out that $c_n(A) \simeq C n^t exp(A)^n$ where $t \in \frac{1}{2}Z$. Later in [16] it was shown that even if A does not have a unit element, still $C_1 n^t exp(A)^n \leq c_n(A) \leq C_2 n^t exp(A)^n$ holds where C_1 and C_2 are positive constants.

When an algebra A has an additional structure, such as a group grading or an involution, one can consider the corresponding codimension sequence and ask if the analogue of the theorem on the existence of the exponent holds.

In this setting it was recently shown that if A is any PI-algebra graded by a finite group then the corresponding exponent exists and is an integer [3,11,1]. Also it turns out that if the algebra is finite dimensional and is acted on by a finite group of automorphisms and antiautomorphisms or by a finite dimensional Lie algebra of derivations or more