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Journal of Algebra

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Normal supercharacter theories and their supercharacters



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ARTICLE INFO

Article history:

Received 22 April 2016

Available online 19 September 2016

Communicated by Jean-Yves Thibon

MSC:

20C12

20E15

05E10

Keywords:

Lattice of normal subgroups

Character theory

Supercharacter theory

ABSTRACT

In order to find a tractable theory to substitute for the wild character theory of the group of $n \times n$ unipotent upper-triangular matrices over a finite field \mathbb{F}_q , André and Yan introduced the notion of supercharacter theory. In this paper, we construct a supercharacter theory from an arbitrary set S of normal subgroups of G . We call such supercharacter theory the normal supercharacter theory generated by S . It is shown that normal supercharacter theories are integral, and a recursive formula for supercharacters of the normal supercharacter theory is provided. Also, we indicate that the superclasses of the normal supercharacter theory generated by all normal subgroups of G are given by certain values on the primitive central idempotents. We study the connection between the finest normal supercharacter theory and faithful irreducible characters. Moreover, an algorithm is presented to construct the supercharacter table of the finest normal supercharacter theory from the character table. Finally, we argue that normal supercharacter theories cannot be obtained by previously known supercharacter theory constructions.

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<http://dx.doi.org/10.1016/j.jalgebra.2016.09.005>
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1. Introduction

Representation theory is undeniably at the heart of many areas of mathematics and science. For example it is a fundamental part of Fourier analysis on groups and a powerful method to study problems in abstract algebra. Unfortunately, the classification of all irreducible representations for some families of groups is intractable. These families of groups are known as wild. For one such family of groups, André [2] and Yan [14] introduced the notion of a supercharacter theory as a new tractable tool that would allow us to still get useful information about the representation theory of the groups. Inspired by André's and Yan's work, Diaconis and Isaacs [6] axiomatized the concept of supercharacter theory for arbitrary groups. A given group can have multiple supercharacter theories. The problem is then to describe, in a useful way, how to obtain such supercharacter theories with enough information about the representations. We present here a new way to construct a supercharacter theory from an arbitrary set of normal subgroups.

Diaconis and Isaacs [6] also mentioned two supercharacter theory constructions for G , one comes from the action of a group A on G by automorphisms and another one comes from the action of cyclotomic field $\mathbb{Q}[\zeta_{|G|}]$. They also generalized André's original construction to define a supercharacter theory for **algebra group**, a group of the form $1 + J$ where J is a finite dimensional nilpotent associative algebra over a finite field \mathbb{F}_q . Later, in [8,9], Hendrickson provided other supercharacter theories for G . Arias-Castro, Diaconis, and Stanley [3] used Yan's work in place of the usual irreducible character theory to study random walks on $UT_n(q)$, the group of $n \times n$ unipotent upper-triangular matrices over a finite field \mathbb{F}_q . Also in [1], the authors obtained a relationship between the supercharacter theory of all unipotent upper-triangular matrices over a finite field \mathbb{F}_q simultaneously and the combinatorial Hopf algebra of symmetric functions in non-commuting variables.

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