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Jordan algebras at Jordan elements of semiprime rings with involution

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ABSTRACT

In this paper we determine the Jordan algebras associated to ad-nilpotent elements of index at most 3 in Lie algebras R^- and $\mathrm{Skew}(R,*)$ for semiprime rings R without or with involution $*$. To do so we first characterize these ad-nilpotent elements.

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1. Introduction

Local algebras at elements were introduced by Meyberg in a nonassociative context [29], and they have also proved to be a very useful tool in the setting of associative systems, see [14]. In the Jordan setting they were used by Zelmanov as a minor part of his brilliant classification of Jordan systems [33–35], and were revisited by D'Amour and McCrimmon in [10]. They have played a prominent role in the structure theory of Jordan systems, mainly due to the fact that nice properties flow between the system and its local algebras. Thus, D'Amour and McCrimmon extended a substantial part of Zelmanov's results to arbitrary quadratic Jordan systems by making use of local algebras [11]. On the other hand, ad-nilpotent elements of index at most 3 (here called Jordan elements) play a fundamental role in the proof of Kostrikin's conjecture that any finite-dimensional simple nondegenerate Lie algebra (over a field of characteristic greater than 5) is classical [8,30]. Jordan elements are also of great importance in the Lie inner ideal structure of associative rings [7].

The analogue of local algebras for Lie algebras was introduced by the second two authors and A. Fernández López in [13]. They showed that it is possible to attach a Jordan algebra to any Jordan element of a Lie algebra. Since their introduction, these Jordan algebras have proven to be very useful: they inherit good properties from the Lie algebra itself, such as nondegeneracy [13], strong primeness [16] and even local finiteness [20], so the structure theory of Jordan systems can be transferred to the original Lie algebra. For example, in [15] the authors revisited the celebrated paper of Zelmanov [32] by using Jordan algebras of Lie algebras. Moreover, Jordan elements have been used as tools in the study of infinite-dimensional Lie algebras. Indeed, they are important in the recent description of simple, infinite-dimensional, locally finite and locally nondegenerate Lie algebras with Jordan elements, see [20, Theorem 1], and A. Baranov and J. Rowley gave a characterization of infinite locally finite simple diagonal Lie algebras in terms of inner ideals, which are closely related to Jordan elements, see [4].

We focus on two types of Lie algebras coming from the associative context: R^- and $\mathrm{Skew}(R,*)$ for centrally closed semiprime rings R. For these Lie algebras, we highlight the recent works [1] and [3]. Our aim is, on the one hand, to describe Jordan elements of semiprime rings, and on the other hand, to describe the Jordan algebras of Lie algebras of the form R^- and $\mathrm{Skew}(R,*)$ for a centrally closed semiprime ring R at those Jordan elements. We characterize those Jordan algebras in terms of local algebras of the original ring (in the case of R^-) and in terms of local algebras of the symmetric Martindale ring of quotients of R for the case with involution.

The first step in this project is to associate a nilpotent element to any Jordan element. Jordan elements are directly associated to a particular case of nilpotent derivations, and this has been a topic of interest since the 1960's. In 1963, I.N. Herstein showed that any ad-nilpotent element a of index n in a simple ring R of characteristic zero or greater than n gives rise to a nilpotent element $a-\lambda$ for some λ in the center of R. Moreover, he showed that the index of nilpotency of such element is less than or equal to $\lfloor \frac{n+1}{2} \rfloor$, see