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On centrally generically tame algebras over perfect fields

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A B S T R A C T

We show that the central generic tameness of a finite-dimensional algebra Λ over a (possibly finite) perfect field, is equivalent to its non-almost sharp wildness. In this case: we give, for each natural number d, parametrizations of the indecomposable Λ-modules with central endolength d, modulo finite scalar extensions, over rational algebras. Moreover, we show that the central generic tameness of Λ is equivalent to its semigeneric tameness, and that in this case, algebraic boundedness coincides with central finiteness for generic Λ-modules.

1. Introduction

Denote by k a fixed ground field and let Λ be a finite-dimensional k-algebra. Given a Λ-module G, recall that by definition the *endolength* G is its length as a right

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$\mathrm{End}_\Lambda(G)^{op}$-module. The module G is called *generic* if it is indecomposable, of infinite length as a Λ-module, but with finite endolength. The algebra Λ is called *generically tame* if, for each $d \in \mathbb{N}$, there is only a finite number of isoclasses of generic Λ-modules with endolength d. This notion was introduced and examined by W.W. Crawley-Boevey in [8] and [9]. In this paper we continue our exploration of the notion of generic tameness for finite-dimensional algebras Λ over perfect fields (see [2–4]). Our main results, stated below, apply to generically tame finite-dimensional algebras Λ over a perfect (possibly finite) field k. In order to state precisely and comment these results we need to recall and introduce some terminology in the following definitions.

Definition 1.1. For any k-algebra B and $M \in B\text{-}\mathrm{Mod}$, denote by $E_M := \mathrm{End}_B(M)^{op}$ its endomorphism algebra. Then, M admits a structure of B-E_M-bimodule. By definition, the *endolength* of M, denoted by $\mathrm{endol}(M)$, is the length of M as a right E_M-module.

A module $M \in B\text{-}\mathrm{Mod}$ is called *pregeneric* iff M is indecomposable, with finite endolength but with infinite dimension over the ground field k. The algebra B is called *pregenerically tame* iff, for each natural number d, there are only finitely many isoclasses of pregeneric B-modules with endolength d.

Definition 1.2. With the preceding notation, given $M \in B\text{-}\mathrm{Mod}$, write $D_M = E_M/\mathrm{rad}\, E_M$ and denote by Z_M the center of D_M. We shall say that the B-module M is *centrally finite* iff D_M is a division ring and $[D_M : Z_M]$ is finite. In this case, $[D_M : Z_M] = c_M^2$, for some positive integer c_M. If M is centrally finite, the *central endolength* of M is the number $c\text{-}\mathrm{endol}(M) = c_M \times \mathrm{endol}(M)$.

The algebra B is called *centrally pregenerically tame*, if for each $d \in \mathbb{N}$ there is only a finite number of isoclasses of centrally finite pregeneric B-modules with central endolength d.

Definition 1.3. Again with the preceding notation, a pregeneric B-module G is called *algebraically rigid* if, for any algebraic field extension $\mathbb{L}$ of k, the $B^{\mathbb{L}}$-module $G^{\mathbb{L}}$ is pregeneric.

We say that a pregeneric B-module G is *algebraically bounded* iff there exists a finite field extension $\mathbb{F}$ of k and a finite sequence of algebraically rigid pregeneric $B^{\mathbb{F}}$-modules $G_1, \ldots, G_n$ such that $G^{\mathbb{F}} \cong G_1 \oplus \cdots \oplus G_n$.

An algebra B is called *semipregenerically tame* if for each $d \in \mathbb{N}$ there is only a finite number of isoclasses of algebraically bounded and centrally finite pregeneric B-modules with central endolength d.

If B is a finite-dimensional algebra, the notion of pregeneric B-module coincides with the usual notion of generic B-module. Hence, in this case we will eliminate the term "pre" which appears in the preceding denominations.

In [4] we obtained for a finite-dimensional semigenerically tame algebra Λ over a perfect field, parametrizations of the centrally finite algebraically bounded Λ-modules