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Applications of the defect of a finitely presented functor

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ABSTRACT

For an abelian category $\mathcal{A}$, the defect sequence

$$0 \longrightarrow F_0 \longrightarrow F \xrightarrow{\varphi} \big(w(F), \_\ \big) \longrightarrow F_1 \longrightarrow 0$$

of a finitely presented functor is used to establish the CoYoneda Lemma. An application of this result is the fp-dual formula which states that for any covariant finitely presented functor F, $F^* \cong \big(\_\ , w(F)\big)$. The defect sequence is shown to be isomorphic to both the double dual sequence

$$0 \longrightarrow \mathsf{Ext}^1(\mathrm{Tr} F, \mathsf{Hom}) \longrightarrow F \longrightarrow F^{**} \longrightarrow \mathsf{Ext}^2(\mathrm{Tr} F, \mathsf{Hom}) \longrightarrow 0$$

and the injective stabilization sequence

$$0 \longrightarrow \overline{F} \longrightarrow F \longrightarrow R^0 F \longrightarrow \tilde{F} \longrightarrow 0$$

establishing the fp-injective stabilization formula $\overline{F} \cong \mathsf{Ext}^1(\mathrm{Tr} F, \mathsf{Hom})$ for any finitely presented functor F. The injectives of $\mathsf{fp}(\mathsf{Mod}(R), \mathsf{Ab})$ are used to compute the left derived functors $L^k(\_\ )^*$. These functors are shown to detect certain short exact sequences in $\mathsf{Mod}(R)$.

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Contents

1. Introduction

For any abelian category $\mathcal{A}$, the category of finitely presented functors $\mathsf{fp}(\mathcal{A}, \mathsf{Ab})$ consists of all functors $F\colon \mathcal{A} \to \mathsf{Ab}$ for which there exists an exact sequence

$$(Y, \_\,) \longrightarrow (X, \_\,) \longrightarrow F \longrightarrow 0$$

One of Auslander's major contributions to representation theory was the demonstration that one may study the category $\mathcal{A}$ by studying the category $\mathsf{fp}(\mathcal{A}, \mathsf{Ab})$ of finitely presented functors. This originates in [1] and is continued in many subsequent works such as [2,3], and [4]. The functorial techniques Auslander used to study finitely presented functors have become widespread in representation theory of algebras but have also been applied to different fields such as algebraic geometry and model theory. For more information, the reader is referred to [11] and [14].

This paper focuses on applications of the defect which is an exact contravariant functor

$$w\colon \mathsf{fp}(\mathcal{A}, \mathsf{Ab}) \longrightarrow \mathcal{A}$$

constructed by Auslander in [1]. Given a finitely presented functor F and a presentation

$$(Y, \_\,) \longrightarrow (X, \_\,) \longrightarrow F \longrightarrow 0$$

the functor w is completely determined by the exact sequence

$$0 \longrightarrow w(F) \longrightarrow X \longrightarrow Y$$

The functor w reveals information at a local level about the functor F and from a more macroscopic point of view it reveals information about the category $\mathcal{A}$. Auslander also associates to each finitely presented functor F, an exact sequence