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Classification of metaplectic modular categories ☆

Eddy Ardonne [a], Meng Cheng [b], Eric C. Rowell [c,*], Zhenghan Wang [d]

[a] *Department of Physics, Stockholm University, Albanova University Center, SE-106 91 Stockholm, Sweden*
[b] *Microsoft Station Q, University of California, Santa Barbara, CA 93106-6105, USA*
[c] *Department of Mathematics, Texas A&M University, College Station, TX 77843, USA*
[d] *Microsoft Station Q and Dept of Mathematics, University of California, Santa Barbara, CA 93106-6105, USA*

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We obtain a classification of metaplectic modular categories: every metaplectic modular category is a gauging of the particle–hole symmetry of a cyclic modular category. Our classification suggests a conjecture that every weakly-integral modular category can be obtained by gauging a symmetry (including the fermion parity) of a pointed (super-)modular category.

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* Corresponding author.
E-mail addresses: ardonne@fysik.su.se (E. Ardonne), mcheng@microsoft.com (M. Cheng), rowell@math.tamu.edu (E.C. Rowell), zhenghwa@microsoft.com (Z. Wang).

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1. Introduction

Achieving a classification of modular categories analogous to the classification of finite abelian groups is an interesting mathematical problem [4,5]. In this note, we classify metaplectic modular categories. Our classification suggests a close connection between finite abelian groups and weakly-integral modular categories via gauging, thus leads to a potential approach to proving the Property F conjecture for weakly-integral modular categories [2,6,14].

A simple object X is weakly-integral if its squared quantum dimension d_X^2 is an integer. A modular category is weakly-integral if every simple object is weakly-integral. Inspired by the applications to physics and topological quantum computation, we focus on weakly-integral modular categories [7,8]. An important class of weakly-integral modular categories is the class of metaplectic modular categories—unitary modular categories with the fusion rules of $SO(N)_2$ for some odd integer $N > 1$ [11,12]. The metaplectic modular categories first appeared in the study of parafermion zero modes, which generalize the Majorana zero modes. The name *metaplectic* comes from the fact that the resulting braid group representations from the generating simple objects in $SO(N)_2$ are the metaplectic representations, which are the symplectic analogues of the spinor representations. Our main result is a classification of metaplectic modular categories: every metaplectic modular category is a gauging of the particle–hole symmetry of a cyclic modular category.

The property F conjecture says that all braid group representations afforded by a weakly-integral simple object have finite images. For $SO(N)_2$, the property F conjecture follows from [15]. It is possible that all weakly-integral modular categories can be obtained by gauging symmetries of pointed modular categories including fermion parities of pointed super-modular categories [3]—categories with all simple objects having their quantum dimension equal to 1. Our classification supports this possibility. If this is true, then a potential approach to the property F conjecture for all weakly-integral modular categories would be to prove that gauging preserves property F.

2. Cyclic modular categories

Definition 2.1. Let $\mathbb{Z}_n$ be the cyclic group of n elements. A $\mathbb{Z}_n$-cyclic modular category is a modular category whose fusion rule is the same as the cyclic group $\mathbb{Z}_n$ for some integer n.

A $\mathbb{Z}_n$-cyclic modular category is determined by a non-degenerate quadratic form $q : \mathbb{Z}_n \to \mathbb{U}(1)$ (see [13] and [10, Appendix D]). We will denote the $\mathbb{Z}_n$-cyclic modular category determined by such a quadratic form q as $\mathcal{C}(\mathbb{Z}_n, k)$ for $q(j) = e^{2\pi i s_j}$, $s_j = \frac{kj^2}{n}$, $0 \le j \le n-1$, $(k, n) = 1$. We will mostly be interested in the case n odd, for which there is always a symmetric bicharacter β such that $q(j) = \beta(j, j)$, from which the braiding on $\mathcal{C}(\mathbb{Z}_n, k)$ is obtained.