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Modules of constant Jordan type, pullbacks of bundles and generic kernel filtrations

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ABSTRACT

Let kE denote the group algebra of an elementary abelian p-group of rank r over an algebraically closed field of characteristic p. We investigate the functors $\mathcal{F}_i$ from kE-modules of constant Jordan type to vector bundles on $\mathbb{P}^{r-1}(k)$, constructed by Benson and Pevtsova. For a kE-module M of constant Jordan type, we show that restricting the sheaf $\mathcal{F}_i(M)$ to a dimension $s-1$ linear subvariety of $\mathbb{P}^{r-1}(k)$ is equivalent to restricting M along a corresponding rank s shifted subgroup of kE and then applying $\mathcal{F}_i$.

In the case $r = 2$, we examine the generic kernel filtration of M in order to show that $\mathcal{F}_i(M)$ may be computed on certain subquotients of M whose Loewy lengths are bounded in terms of i. More precise information is obtained by applying similar techniques to the nth power generic kernel filtration of M. The latter approach also allows us to generalise our results to higher ranks r.

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1. Introduction

The goal of this paper is to further investigate a curious functorial relationship between the category of finitely generated kE-modules and the category of coherent sheaves on the projective space $\mathbb{P}^{r-1}(k)$, where E is an elementary abelian p-group of rank r and k is an algebraically closed field of characteristic p. Specifically, we wish to better understand the functors

$$\mathcal{F}_i\colon\ \mathsf{mod}(kE) \longrightarrow \mathsf{coh}(\mathbb{P}^{r-1}(k)), \qquad 1 \leq i \leq p$$

introduced by Benson and Pevtsova [4]. Interest in the functors $\mathcal{F}_i$ originated in the study of kE-modules of constant Jordan type, which were defined by Carlson, Friedlander and Pevtsova [5]. Denoting the subcategory of modules of constant Jordan type by $\mathsf{cJt}(kE)$, Benson and Pevtsova showed that the functors $\mathcal{F}_i$ descend to functors

$$\mathcal{F}_i\colon\ \mathsf{cJt}(kE) \longrightarrow \mathsf{vec}(\mathbb{P}^{r-1}(k)),$$

where $\mathsf{vec}(\mathbb{P}^{r-1}(k))$ is the category of vector bundles on $\mathbb{P}^{r-1}(k)$. We remark that neither of the latter two categories is well understood. Whereas the study of modules of constant Jordan type is a relatively new enterprise, the attempt to understand what sorts of vector bundles can live on $\mathbb{P}^{r-1}(k)$ has been ongoing since the advent of modern algebraic geometry, and with limited success. Accordingly, a thorough understanding of the functors $\mathcal{F}_i$ should be of interest to representation theorists and algebraic geometers alike.

In this direction, our aim is to further establish some sort of dictionary between modules of constant Jordan type and vector bundles on $\mathbb{P}^{r-1}(k)$ via the functors $\mathcal{F}_i$. For example, one of the common techniques of the algebraic geometer is that of restricting a vector bundle on $\mathbb{P}^{r-1}(k)$ to a line L in $\mathbb{P}^{r-1}(k)$ in order to compute its so called 'splitting type'. Any such closed immersion $L \subseteq \mathbb{P}^{r-1}(k)$ is obtained by applying the Proj functor to a surjective homogeneous ring homomorphism