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On the vanishing of local cohomology of the absolute integral closure in positive characteristic ☆

Pham Hung Quy

Department of Mathematics, FPT University, Hoa Lac Hi-Tech Park, Ha Noi, Viet Nam

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The aim of this paper is to extend the main result of C. Huneke and G. Lyubeznik in [Adv. Math. 210 (2007), 498–504] to the class of rings that are images of Cohen–Macaulay local rings. Namely, let R be a local Noetherian domain of positive characteristic that is an image of a Cohen–Macaulay local ring. We prove that all local cohomology of R (below the dimension) maps to zero in a finite extension of the ring. As a direct consequence we obtain that the absolute integral closure of R is a big Cohen–Macaulay algebra. Since every excellent local ring is an image of a Cohen–Macaulay local ring, this result is a generalization of the main result of M. Hochster and Huneke in [Ann. of Math. 135 (1992), 45–79] with a simpler proof.

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E-mail address: quyph@fpt.edu.vn.

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1. Introduction

Let $(R, \mathfrak{m})$ be a commutative Noetherian local domain with fraction field K. The *absolute integral closure* of R, denoted R^+, is the integral closure of R in a fixed algebraic closure $\overline{K}$ of K.

A famous result of M. Hochster and C. Huneke says that if $(R, \mathfrak{m})$ is an excellent local Noetherian domain of positive characteristic $p > 0$, then R^+ is a (balanced) big Cohen–Macaulay algebra, i.e. every system of parameters in R becomes a regular sequence in R^+ (cf. [7]). Furthermore, K.E. Smith in [15] proved that the tight closure of an ideal generated by parameters is the contraction of its extension in R^+: $I^* = IR^+ \cap R$. This property is not true for every ideal I in an excellent Noetherian domain since tight closure does not commute with localization (cf. [1]).

As mentioned above, $H^i_{\mathfrak{m}}(R^+) = 0$ for all $i < \dim R$ provided R is an excellent local Noetherian domain of positive characteristic. Hence, the natural homomorphism $H^i_{\mathfrak{m}}(R) \to H^i_{\mathfrak{m}}(R^+)$, induced from the inclusion $R \to R^+$, is the zero map for all $i < \dim R$. In the case R is an image of a Gorenstein (not necessarily excellent) local ring, as the main result of [8], Huneke and G. Lyubeznik proved a stronger conclusion that one can find a finite extension ring S, $R \subseteq S \subseteq R^+$, such that the natural map $H^i_{\mathfrak{m}}(R) \to H^i_{\mathfrak{m}}(S)$ is zero for all $i < \dim R$. Therefore, they obtained a simpler proof for the result of Hochster and Huneke in the cases where the assumptions overlap, e.g., for complete Noetherian local domain. The techniques used in [8] are the Frobenius action on the local cohomology, (modified) equation lemma (cf. [7,15,8]) and the local duality theorem (this is the reason of the assumption that R is an image of a Gorenstein local ring). The motivation of the present paper is our belief: *If a result was shown by the local duality theorem, then it can be proven under the assumption that the ring is an image of a Cohen–Macaulay local ring* (for example, see [12]). The main result of this paper extends Huneke–Lyubeznik's result to the class of rings that are images of Cohen–Macaulay local rings. Namely, we prove the following.

Theorem 1.1. *Let $(R, \mathfrak{m})$ be a commutative Noetherian local domain containing a filed of positive characteristic p. Let K be the fraction field of R and $\overline{K}$ an algebraic closure of K. Assume that R is an image of a Cohen–Macaulay local ring. Let R' be an R-subalgebra of $\overline{K}$ (i.e. $R \subseteq R' \subseteq \overline{K}$) that is a finite R-module. Then there is an R'-subalgebra R'' of $\overline{K}$ (i.e. $R' \subseteq R'' \subseteq \overline{K}$) that is finite as an R-module such that the natural map $H^i_{\mathfrak{m}}(R') \to H^i_{\mathfrak{m}}(R'')$ is the zero map for all $i < \dim R$.*

As a direct application of Theorem 1.1 we obtain that the absolute integral closure R^+ is a big Cohen–Macaulay algebra (cf. Corollary 3.2). It worth be noted that every excellent local ring is an image of a Cohen–Macaulay excellent local ring by T. Kawasaki (cf. [9, Corollary 1.2]). Therefore, our results also extend the original result of Hochster and Huneke with a simpler proof. The main results will be proven in the last section. In

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