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Hasse principle for hermitian spaces over semi-global fields

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ABSTRACT

In a recent paper, Colliot-Thélène, Parimala and Suresh conjectured that a local–global principle holds for projective homogeneous spaces under connected linear algebraic groups over function fields of p-adic curves. In this paper, we show that the conjecture is true for any linear algebraic group whose almost simple factors of its semisimple part are isogenous to unitary groups or special unitary groups of hermitian or skew-hermitian spaces over central simple algebras with involutions. The proof implements patching techniques of Harbater, Hartmann and Krashen. As an application, we obtain a Springer-type theorem for isotropy of hermitian spaces over odd degree extensions of function fields of p-adic curves.

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1. Introduction

Let T be a complete discrete valuation ring with residue field k. Let K be the field of fractions of T. Let F be the function field of a smooth, projective, geometrically integral curve $\mathscr{X}_0$ over K. Recently, such a field F has been called a *semi-global* field. Let Ω be the set of all rank one discrete valuations on F (or the set of all divisorial discrete valuations from all codimension one points of all regular projective models $\mathscr{X} \to \mathrm{Spec}(T)$ of the curve $\mathscr{X}_0$). For each $v \in \Omega$, let F_v be the completion of F at v. Let G be a connected linear algebraic group over F and let X be a projective homogeneous space under G over F. We say that the *Hasse principle holds* for X if

$$\prod_{v\in\Omega} X(F_v) \neq \emptyset \implies X(F) \neq \emptyset.$$

Colliot-Thélène, Parimala and Suresh [16, 3.1] have proved that if $\mathrm{char}\, k \neq 2$ and $q : V \to F$ is a quadratic form with $\dim_F(V) \geq 3$, then the Hasse principle holds for every projective homogeneous space under $\mathrm{SO}(q)$. Reddy and Suresh [38, 2.6] have proved that if A is a central simple F-algebra of degree coprime to $\mathrm{char}\, k$, then the Hasse principle holds for every projective homogeneous space under $\mathrm{PGL}_1(A)$. After [15, 3.1] and [14, 5.7], Harbater, Hartmann and Krashen [19, 9.2] have proved that if k is algebraically closed and $\mathrm{char}\, k = 0$, then the Hasse principle holds for projective homogeneous spaces under connected *rational* groups.

In this article we explore the Hasse principle for projective homogeneous spaces under G over F for certain groups of classical types.

Assumption 1.1. Suppose K, k, F, Ω, G, X are as before and $\mathrm{char}\, k \neq 2$. Let A be a finite-dimensional simple associative F-algebra with an involution σ such that $F = Z(A)^\sigma$. Let $h : V \times V \to A$ be an ε-hermitian space over (A, σ) for $\varepsilon \in \{1, -1\}$. Let

$$G = \begin{cases} \mathrm{SU}(A, \sigma, h), & \text{if } \sigma \text{ is of the first kind;} \\ \mathrm{U}(A, \sigma, h), & \text{if } \sigma \text{ is of the second kind.} \end{cases}$$