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On Mitchell's embedding theorem for a quasi-schemoid

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ABSTRACT

A quasi-schemoid is a small category whose morphisms are colored with appropriate combinatorial data. In this paper, Mitchell's embedding theorem for a *tame* schemoid is established. The result allows us to give a cofibrantly generated model category structure to the category of chain complexes over a functor category with a schemoid as the domain. Moreover, a notion of Morita equivalence for schemoids is introduced and discussed. In particular, we show that every Hamming scheme of binary codes is Morita equivalent to the association scheme arising from the cyclic group of order two. In an appendix, we construct a new schemoid from an abstract simplicial complex, whose Bose–Mesner algebra is closely related to the Stanley–Reisner ring of the given complex.

1. Introduction

There are two crucial categories for representation theory of small categories including groups and quivers. One is a module category and another one is a functor category.

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Mitchell's embedding theorem [13, Theorem 7.1] states that these categories are equivalent provided the small category, which we deal with, has finite many objects.

Association schemes, ASs for short, are significant subjects in algebraic combinatorics; see [1,5,19]. These subjects give rise to the so-called Bose–Mesner algebras (adjacency algebras) and the study of the algebras creates applications in the theory of codes and designs; see for example [18]. An important point is that the category of finite groups is embedded in the category of ASs in the sense of Hanaki; see [20,7,6]. If an association scheme is thin (in the sense of [20]), then its Bose–Mesner algebra is just the group ring of the corresponding group. Thus, representation theory of ASs is developed in the module categories of their Bose–Mesner algebras. However, until today there are few studies on ASs dealing with their categorical and homological structures such as group cohomology.

Very recently, Matsuo and the first author [11] have introduced the notion of *quasi-schemoids* generalizing that of ASs from a small categorical point of view. Roughly speaking, a quasi-schemoid is a small category whose morphisms are colored with appropriate combinatorial data. In [15], Momose has considered representation theory for quasi-schemoids with module categories of their Bose–Mesner algebras. In this manuscript, we develop *another* representation theory, namely that based on an appropriate functor category with a quasi-schemoid as the domain. It is worthwhile to remark that the two categories for representation theory of schemoids are not equivalent in general even the set of objects in the underlying category of a given quasi-schemoid is finite. That is, Mitchell's embedding theorem does not necessarily hold in our context; see Proposition 4.3 and Remark 4.5.

One of the aims of this manuscript is to give a class of schemoids in which Mitchell's embedding theorem holds; see Theorem 2.4. Such schemoids are called *tame*. Our functor category for a schemoid is a subcategory, but not full, of the usual one for the underlying category. Therefore, the existence of left and right adjoints to a restriction functor is not immediate. We will also discuss this problem; see Theorem 2.6.

An outline for the article is as follows. In Section 2, we describe our main theorems concerning Mitchell's embedding and adjoint functors on functor categories of schemoids. By employing the adjoint pair, we define a cofibrantly generated model category structure on the category of chain complexes over a functor category with a schemoid as the domain; see Theorem 2.8. Moreover, schemoid cohomology of a morphism between schemoids and a notion of Morita equivalence of schemoids are introduced. In Section 3, after defining a tame schemoid explicitly, we prove our main theorems. Section 4 concerns examples of schemoid cohomology and a Morita equivalence. In particular, we shall show that every Hamming scheme of binary codes is Morita equivalent to the association scheme arising from the cyclic group of order two; see Proposition 4.3. Section 5 explores an invariant for Morita equivalence which is induced by a functor between underlying categories.

In Appendix A, we construct a new schemoid from an abstract simplicial complex, whose schemoid cohomology is investigated in Section 4. This subject is very interesting