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Computing global dimension of endomorphism rings via ladders ☆

Brandon Doherty [a], Eleonore Faber [b,*,1], Colin Ingalls [a]

[a] *Department of Mathematics and Statistics, University of New Brunswick, Fredericton, NB. E3B 5A3, Canada*
[b] *Fakultät für Mathematik, Universität Wien, Oskar-Morgenstern-Platz 1, A-1090 Wien, Austria*

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ABSTRACT

This paper deals with computing the global dimension of endomorphism rings of maximal Cohen–Macaulay (= MCM) modules over commutative rings. Several examples are computed. In particular, we determine the global spectra, that is, the sets of all possible finite global dimensions of endomorphism rings of MCM-modules, of the curve singularities of type A_n for all n, D_n for $n \leq 13$ and $E_{6,7,8}$ and compute the global dimensions of Leuschke's normalization chains for all ADE curves, as announced in [12]. Moreover, we determine the centre of an endomorphism ring of a MCM-module over any curve singularity of finite MCM-type.

In general, we describe a method for the computation of the global dimension of an endomorphism ring $\operatorname{End}_R M$, where R is a Henselian local ring, using $\operatorname{add}(M)$-approximations. When $M \neq 0$ is a MCM-module over R and R is Henselian local of Krull dimension ≤ 2 with a canonical module and of finite

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* Corresponding author.
E-mail addresses: Brandon.Doherty@unb.ca (B. Doherty), emfaber@umich.edu (E. Faber), cingalls@unb.ca (C. Ingalls).
[1] *Current address*: Department of Mathematics, University of Michigan, Ann Arbor, MI 48109, USA.

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MCM-type, we use Auslander–Reiten theory and Iyama's ladder method to explicitly construct these approximations.

Contents

1. Introduction

Let R be a commutative noetherian ring, $M \neq 0$ be a MCM-module over R and set $A = \operatorname{End}_R M$. This article is concerned with the problem of computing the global dimension of A, that is, the smallest number n such that any A-module has a projective resolution of length $\leq n$. In most cases, n will be infinity: for example, if A itself is commutative, then $\operatorname{gl.dim} A < \infty$ if and only if A is a regular ring, by Serre's well-known theorem. But if A is noncommutative, the situation is much more involved and one is precisely interested in this case: in recent years the study of endomorphism rings of finite global dimension has become increasingly popular, since they appear as analogues of resolutions of singularities of $\operatorname{Spec}(R)$, namely as so-called noncommutative (crepant) resolutions of singularities (=NC(C)Rs). In the original treatment of van den Bergh [41] NCCRs over Gorenstein normal domains R were defined as homologically homogeneous endomorphism rings of finitely generated reflexive R-modules, which implies that their global dimension is automatically equal to the Krull dimension of R. NCCRs were further studied in, e.g., [10,23] and see [32] for an overview. In [15] and [12], more general NCRs of any commutative ring R were defined as endomorphism rings of finitely generated modules of full support and of finite global dimension. In [12] the *global spectrum* of R