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Embeddings of Alt_n and its perfect covers for $n \geq 6$ in exceptional complex Lie groups

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A R T I C L E I N F O

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A B S T R A C T

We classify Alt_n and its perfect covers for $n \geq 6$ up to conjugacy as subgroups of the exceptional complex Lie groups (with some exceptions where we give lower bounds and discuss the possibilities of Lie primitive subgroups).

1. Statement of main results

In Griess and Ryba's 2002 paper [15], all finite quasisimple groups that embed in the exceptional complex Lie groups were classified up to isomorphism. However, conjugacy results are few and far between. In a series of papers ([7–9]), this author classified embeddings of the alternating group Alt_5 and its double cover $SL(2, 5)$ up to conjugacy in the exceptional complex Lie groups other than $G_2(\mathbb{C})$ except for a couple of cases where the subgroup centralizers are zero-dimensional. (Embeddings of finite subgroups in $G_2(\mathbb{C})$ are elegantly classified up to conjugacy in [13].) Since our methods depend heavily on centralizers, these zero-dimensional centralizer cases required a different approach. Such

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[1] Appendix A by A.J.E. Ryba.

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Table 1
The main theorem.

Isomorphism class	Conjugacy classes of groups (embeddings) in $F_4(\mathbb{C})$	$3^{\cdot}E_6(\mathbb{C})$	$2^{\cdot}E_7(\mathbb{C})$	$E_8(\mathbb{C})$
Alt_6	$\geq 3(\geq 5)$	$\geq 6(\geq 10)$	$\geq 7(\geq 12)$	$\geq 11(\geq 18)$
Alt_7	x	1(1)	1(1)	$\geq 2(\geq 2)$
Alt_8	x	x	1(1)	1(1)
Alt_9	x	x	2(3)	3(4)
Alt_{10}	x	x	x	$\geq 1(\geq 1)$
$2^{\cdot}Alt_6$	1(2)	2(3)	2(3)	9(17)
$2^{\cdot}Alt_7$	1(1)	1(1)	1(1)	4(4)
$2^{\cdot}Alt_8$	1(1)	1(1)	1(1)	3(3)
$2^{\cdot}Alt_9$	1(1)	1(1)	1(1)	$\geq 3(\geq 4)$
$2^{\cdot}Alt_{10}$	1(1)	1(1)	1(1)	$\geq 2(\geq 2)$
$2^{\cdot}Alt_{11}$	x	1(1)	1(1)	1(1)
$2^{\cdot}Alt_{12}$	x	x	1(1)	1(1)
$2^{\cdot}Alt_{13}$	x	x	1(1)	1(1)
$2^{\cdot}Alt_{14}$	x	x	x	1(1)
$2^{\cdot}Alt_{15}$	x	x	x	1(1)
$2^{\cdot}Alt_{16}$	x	x	x	1(1)
$2^{\cdot}Alt_{17}$	x	x	x	1(1)
$3^{\cdot}Alt_6$	4(8)	9(22)	12(22)	$\geq 17(\geq 34)$
$3^{\cdot}Alt_7$	x	1(2)	2(2)	1(1)
$6^{\cdot}Alt_6$	x	1(4)	4(12)	4(12)
$6^{\cdot}Alt_7$	x	1(4)	2(4)	1(2)

an approach was suggested by J.P. Serre and accomplished by Lusztig in [18]. In this paper, we expand our results to encompass all of the alternating groups and their nonsplit central extensions. Again, while our methods were able to handle most cases, there are some zero-dimensional centralizer cases that will require a different approach.

Theorem 1.1. *Table 1 lists the numbers of conjugacy classes of subgroups and embeddings of alternating groups and their covers in exceptional complex Lie groups. Table 2 lists examples of such embeddings which factor through a centralizer subgroup.*

For each entry in Table 1, the first number represents the number of conjugacy classes of subgroups for that particular isomorphism type in the appropriate exceptional complex Lie group. The number in parentheses represents the number of conjugacy classes of embeddings. An "x" means there are no embeddings. In some cases, we were only able to establish a lower bound on the number of classes. These cases are indicated in Table 1 with a "$\geq$."

Almost all of the subgroups in the main theorem factor through element centralizers. In Table 2, we indicate the embedding in the centralizer subgroup through which the given $r^{\cdot}Alt_n$ subgroup factors. In each case, the embedding is represented by a character followed by the name of a positive dimensional subgroup in parentheses (such as (B), (Φ), $(\mathcal{D})$) which means the $r^{\cdot}Alt_n$ subgroup lies in a connected subgroup (B, Φ, $\mathcal{D}$, to be defined in the next section) via a representation corresponding to the given character. We indicate those cases where there is the possibility of a corresponding Lie primitive subgroup (i.e. one that is not contained in a proper, connected subgroup) with an LP superscript. (If there are too many cases for an entry in Table 2, we refer the reader to