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Splendid and perverse equivalences

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A B S T R A C T

Inspired by the works of Rickard on splendid equivalences [9] and of Chuang and Rouquier on perverse equivalences [6], we are here interested in the combination of both, *i.e.* a splendid perverse equivalence. This is naturally the right framework to understand the relations between global and local perverse equivalences between blocks of finite groups, as a splendid equivalence induces local derived equivalences via the Brauer functor. We prove that under certain conditions, we have an equivalence between a perverse equivalence between the homotopy category of p-permutation modules and local derived perverse equivalences, in the case of abelian defect groups.

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0. Motivation and notation

We here work on the classical dynamic of global and local properties for the representation theory of a finite group G. We recall that by global one means a finite group G while by local one means working at the level of centralizers (or normalizers) of p-subgroups of G.

A splendid complex clearly restricts to an equivalence between the homotopy categories of p-permutation modules. Surprisingly, the converse also holds. However if one was to add that the equivalences are moreover perverse, we only have that a perverse equivalence at the level of the homotopy categories of p-permutation modules implies a derived perverse equivalence.

We shall see that in order to obtain local perverse derived equivalences, one should start with the stronger condition of a perverse homotopic equivalence on the homotopy category of p-permutation modules.

As one would hope to realize the global-local connection in a commutative diagram, we shall be working in the homotopy category of p-permutation modules rather than in the corresponding derived category. That way, we shall see that from a global perverse homotopy equivalence, one obtains local derived perverse equivalences. In our attempt to reconstruct the global data from the local data, we will introduce the refined notion of perverse equivalence relative to a partial order.

Finally, we will illustrate this by a careful study of the cyclic case, and see that Rouquier's splendid complex [10] does not necessarily realize a global perverse equivalence although it always induces locally perverse equivalences. Last but not least, we will make precise the connection between the local perversities of Rouquier's complex and the generalised decomposition numbers of the block.

The aim of Section 1 is to develop this theme of global versus local along Boltje and Xu's notion of p-permutation equivalence. In Section 2, we will make precise a result of Rickard on splendid complexes connecting a splendid tilting complex X with its image by the Brauer functor $\mathrm{Br}_{\Delta Q}(X)$. In Section 3, we make the connection between perverse equivalences at the level of the centralizer of a p-group and at the level of the corresponding normalizer. Then in Section 4, we show that a global perverse homotopy equivalence gives rise to local derived perverse equivalence. We then introduce the notion of perverse equivalence relative to a partial order in order to go back up, from the data of local perverse derived equivalence, to a global perverse homotopy equivalence. Then, we gather all of the above results and prove our main result.