Journal of Algebra 450 (2016) 384–397

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Ordered regular equivalence relations on ordered semihypergroups

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ARTICLE INFO

Article history:
Received 12 May 2015
Available online 6 December 2015
Communicated by Gernot Stroth

MSC:
20N20
06F05

Keywords:
Ordered semihypergroup
Ordered regular equivalence relation
Strongly ordered regular equivalence relation
Hyperideal
Hyperfilter
Ordered semigroup

ABSTRACT

In this paper, we introduce the concept of ordered regular (strongly ordered regular) equivalence relation on an ordered semihypergroup. Moreover, we use hyperfilters to construct a strongly ordered regular equivalence relation on an ordered semihypergroup, for which the corresponding quotient structure is a semilattice. Finally, we construct an ordered regular equivalence relation on an ordered semihypergroup by hyperideals such that the corresponding quotient structure is also an ordered semihypergroup under a related order relation, which is an answer to the open problem given by B. Davvaz, P. Corsini and T. Changphas in European Journal of Combinatorics.

1. Introduction

The algebraic hyperstructure theory was introduced in 1934 by Marty [22]. Hyperstructures have many applications in several branches of both pure and applied sciences [5,6,11]. In particular, a semihypergroup is the simplest algebraic hyperstructure which

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http://dx.doi.org/10.1016/j.jalgebra.2015.11.026

is a generalization of the concept of a semigroup. Nowadays many authors have studied different aspects of semihypergroups, for instance, Bonansinga and Corsini [2], Corsini [4], Davvaz [7], Davvaz and Poursalavati [10], De Salvo et al. [12], Fasino and Freni [13], Gutan [15], Hila et al. [17,18], Leoreanu [21], and many others. Regular equivalence relations and strongly regular equivalence relations on semihypergroups were introduced and studied in [5,8]. The homomorphism theory of hyperstructures was first introduced by Krasnerin 1952 and then studied by Massouros [23,25], Mattes and Konstantinidou [30]. Moreover, the application of semihypergroups which is the representation of convex sets by semihypergroups was given by Massouros in [24,26,27].

The first book [14] on ordered algebraic structures was written by Fuchs in 1960's. Many authors, especially Satyanarayana [33], Blyth and Janowitz [1], Kehayopulu and Tsingelis [19,20] and Xie [35,36], studied different aspects of ordered semigroups. The concept of a pseudoorder on an ordered semigroup was introduced and studied by Kehayopulu and Tsingelis [19,20]. Xie [35] introduced and characterized regular, strongly regular congruences on ordered semigroups, which are such that the corresponding quotient structures are also ordered semigroups and satisfy other good properties. In [16], Heidari and Davvaz applied the hyperstructure theory to ordered semigroups and introduced the concept of ordered semihypergroups, which is a generalization of the concept of ordered semigroups. The hyperideals and hyperfilters of ordered semihypergroups were introduced by Changphas and Davvaz [3] and Tang et al. [34] respectively. Recently, Davvaz et al. [9] extended the concept of a pseudoorder to an ordered semihypergroup and used it to construct a strongly regular equivalence relation on an ordered semihypergroup for which the corresponding quotient structure is an ordered semigroup. The fuzzy hyperideals of ordered semihypergroups were studied by Pibaljommee and Davvaz in [32]. Tang et al. also investigated the fuzzy hyperfilters of ordered semihypergroups in [34]. Furthermore, the hyperstructures were enriched with order relations by the work of Mittas, for example [28–31].

This paper is organized as follows. In Section 2, we recall some definitions and results of ordered semihypergroups which will be used in this paper, and introduce the concept of ordered regular (strongly ordered regular) equivalence relation on an ordered semihypergroup. In Section 3, we use hyperfilters to construct a strongly ordered regular equivalence relation on an ordered semihypergroup, for which the corresponding quotient structure is a semilattice. In Section 4, we construct an ordered regular equivalence relation on an ordered semihypergroup by hyperideals such that the corresponding quotient structure is also an ordered semihypergroup, which answers the open problem proposed by Davvaz, Corsini and Changphas in [9].

2. Preliminaries and notations

A *hypergroupoid* $(S, \circ)$ is a nonempty set S together with a *hyperoperation*, that is a mapping $\circ : S \times S \to \mathcal{P}^*(S)$, where $\mathcal{P}^*(S)$ denotes the family of all nonempty subsets