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Torsion pairs in a triangulated category generated by a spherical object

Raquel Coelho Simões [a], David Pauksztello [b,*]

[a] *Centro de Análise Funcional, Estruturas Lineares e Aplicações, Faculdade de Ciências da Universidade de Lisboa, Campo Grande, Edifício C6, Piso 2, 1749-016, Lisboa, Portugal*
[b] *School of Mathematics, The University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom*

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ABSTRACT

We extend Ng's characterisation of torsion pairs in the 2-Calabi–Yau triangulated category generated by a 2-spherical object to the characterisation of torsion pairs in the w-Calabi–Yau triangulated category, T_w, generated by a w-spherical object for any $w \in \mathbb{Z}$. Inspired by the combinatorics of T_w for $w \leqslant -1$, we also characterise the torsion pairs in certain negative Calabi–Yau orbit categories of the bounded derived category of the path algebra of Dynkin type A.

Contents

* Corresponding author.
E-mail addresses: rcoelhosimoes@campus.ul.pt (R. Coelho Simões), david.pauksztello@manchester.ac.uk (D. Pauksztello).

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Introduction

Calabi–Yau (CY) triangulated categories are triangulated categories that satisfy an important duality. They are becoming increasingly important throughout mathematics and physics, for example as 3-CY categories arising from Calabi–Yau threefolds in algebraic geometry and string theory, to 3-CY categories arising in representation theory coming from quivers with potential. Of particular importance in representation theory are (2-)cluster categories, which provide categorifications of important aspects of the theory of cluster algebras. There are higher analogues, so-called w-cluster categories for $w \geqslant 2$, which are w-CY. These give rise to an important family of categories of positive CY dimension which satisfy many interesting and important homological and combinatorial properties.

Throughout this article $\mathbf{k}$ will be an algebraically closed field. Let T be a $\mathbf{k}$-linear triangulated category and $w \in \mathbb{Z}$. An object $s \in \mathsf{T}$ is *w-spherical* if it is a w-Calabi–Yau object and its graded endomorphism algebra is given by

$$\mathsf{Hom}^\bullet(s, s) = \mathbf{k}[x]/(x^2), \text{ where } x \text{ sits in cohomological degree } -w.$$

In particular, s has the 'same cohomology' as the w-sphere. We refer the reader to Section 2 for a more precise definition.

Let T_w be a $\mathbf{k}$-linear triangulated category that is idempotent complete and generated by a w-spherical object. The T_w constitute a family of categories which are w-CY whose structure is sufficiently simple to allow concrete computation. As such, they provide a 'natural laboratory' in which to explore the properties of CY triangulated categories, as witnessed by the intense recent interest in these categories; see [13,15,20,26,28]. Indeed, for $w \geqslant 2$, T_w occurs naturally as a w-cluster category of type A_∞.

Owing to their importance and ubiquity, much work has been carried out on understanding triangulated categories of positive CY-dimension. However, very little work has been carried out on understanding the properties of triangulated categories of negative CY-dimension, although there is the beginning of a theory emerging in [10,12,13,24]. In [24], it was shown that for $w \geqslant 1$, the category T_w has one family of bounded