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Gonosomal algebra

Richard Varro [a,b,*]

[a] Institut de Mathématiques et de Modèlisation de Montpellier, Université de Montpellier, 35095 Montpellier Cedex 5, France
[b] Département de Mathématiques et Informatique Appliquées, Université de Montpellier III, 34199 Montpellier Cedex 5, France

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ABSTRACT

We introduce the gonosomal algebra. Gonosomal algebra extends the evolution algebra of the bisexual population (EABP) defined by Ladra and Rozikov. We show that gonosomal algebras can represent algebraically a wide variety of sex determination systems observed in bisexual populations. We illustrate this by about twenty genetic examples, most of these examples cannot be represented by an EABP. We give seven algebraic constructions of gonosomal algebras, each is illustrated by genetic examples. We show that unlike the EABP gonosomal algebras are not dibaric. We approach the existence of dibaric function and idempotent in gonosomal algebras.

1. Introduction

To construct an algebraic model of sex-linked inheritance (i.e. controlled by sex), it is necessary to solve two problems: the asymmetry of the genes transmission and the wide variety of sex-determination systems.

* Correspondence to: Département de Mathématiques et Informatique Appliquées, Université de Montpellier III, 34199 Montpellier Cedex 5, France.
E-mail address: richard.varro@univ-montp3.fr.

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In populations with separate sexes, bisexual and diploid, the sexual differentiation is controlled in most cases by a pair of chromosomes: the sex chromosomes also called gonosomes (or heterochromosomes, or heterosomes). The gonosomes often have different shapes (dimorphism) which induce an asymmetry in the transmission of sex-linked traits.

In the animal kingdom, sex is determined by different systems, we distinguish five systems: XY, WZ, $X0$, $Z0$ and WXY, which can be added to their multiple variants.

In the XY sex-determination system observed in most mammals, homogametic XX individuals are females and heterogametic XY are males. The XY-system has multiple variants as X_1X_2Y and XY_1Y_2. In the X_1X_2Y-system, females are $X_1X_1X_2X_2$ and males X_1X_2Y while in the system XY_1Y_2 females are XX and males XY_1Y_2. Systems with more gonosomes as $X_1X_2X_3X_4X_5Y$ or $X_1X_2X_3X_4X_5Y_1Y_2Y_3Y_4Y_5$ have also been observed.

The WZ-system is mainly found in birds, females are heterogametic WZ while males are homogametic ZZ. This system also accepts multiple variants such as W_1W_2Z, WZ_1Z_2, $W_1W_2Z_1Z_2$.

The $X0$-system is mainly observed in Hymenoptera (bees, wasps, ...). In this system sex is controlled by a single X chromosome, females have two X chromosomes, their genotype is XX, while males have one, they are $X0$ (zero indicates the absence of the second X chromosome). It has been observed in several species of spiders the following systems X_1X_20, $X_1X_2X_30$, $X_1X_2X_30$, $X_1X_2X_3X_40$.

As for $Z0$ sex determination system mainly observed among Lepidoptera, females are $Z0$ and males ZZ.

Finally, the WXY-system observed in several species of tropical fish is more complex: an individual having a gonosome Y is male unless it is coupled with a W chromosome, so in this system individuals with genotypes XY or YY are males and those of genotypes XX, WX and WY are females.

However we will see later in this work through numerous examples that there are many other sex determination systems.

A gene is said to be sex-linked or gonosomal if it is located on the sex chromosomes. Because of the dimorphism of gonosomes there are two kinds of gonosomal genes. Indeed, in the XY and WZ systems two parts on gonosomes are observed: one part homologous (or pseudo-autosomal) where genes are common to both gonosome types and a differential part where a locus located on a gonosome has no counterpart on the other. There are therefore two types of gonosomal genes: a gene is pseudo-autosomal (or partially sex-linked) if the locus is located on the homologous parts; it is gonosomal (or completely sex-linked) when the locus is on the differential part of the sex chromosomes (cf. [10]).

In this work, after recalling the algebraic models of sex-linked inheritance, we give a genetic example that cannot be represented by these algebras, this leads us to extend these algebras and define the gonosomal algebras. Then we give seven algebraic constructions of gonosomal algebras and illustrating them with examples we see that these algebras can represent algebraically a wide variety of genetic phenomena related to sex as: *i)* temperature-dependent sex determination; *ii*) sequential hermaphrodism;