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Global resolution of singularities subordinated to a 1-dimensional foliation

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ABSTRACT

Let M be an analytic manifold over $\mathbb{R}$ or $\mathbb{C}$, θ a 1-dimensional Log-Canonical (resp. monomial) singular distribution and $\mathcal{I}$ a coherent ideal sheaf defined on M. We prove the existence of a resolution of singularities for $\mathcal{I}$ that preserves the Log-Canonicity (resp. monomiality) of the singularities of θ. Furthermore, we apply this result to provide a resolution of a family of ideal sheaves when the dimension of the parameter space is equal to the dimension of the ambient space minus one.

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1. Introduction

A *foliated ideal sheaf* is a quadruple $(M, \theta, \mathcal{I}, E)$, where: M is a smooth analytic manifold of dimension n over a field $\mathbb{K}$ (where $\mathbb{K}$ is $\mathbb{R}$ or $\mathbb{C}$); $\mathcal{I}$ is a coherent and everywhere non-zero ideal sheaf of M; E is an ordered collection $E = (E^{(1)}, \dots, E^{(l)})$, where each $E^{(i)}$ is a smooth divisor on M such that $\sum_i E^{(i)}$ is a reduced divisor with simple normal crossings (we will say that E is a SNC divisor for shortness); θ is an involutive singular distribution defined over M and everywhere tangent to E. In the same notation, a *foliated analytic manifold* is the triple (M, θ, E).

The main objective of this work is to find a resolution of singularities for $\mathcal{I}$ that preserves the class of singularities of θ. In order to be precise and set notation we briefly recall some basic notions of singular distributions and resolution of singularities:

- **Singular distributions** (we follow [1]): Let Der_M denote the sheaf of analytic vector fields over M, i.e., the sheaf of analytic sections of TM. An *involutive singular distribution* is a coherent sub-sheaf θ of Der_M such that for each point p in M the stalk $\theta_p := \theta.\mathcal{O}_p$ is closed under the Lie bracket operation.
  Consider the quotient sheaf $Q = Der_M/\theta$. The *singular set* of θ is defined by the closed analytic subset $S(\theta) = \{p \in M : Q_p \text{ is not a free } \mathcal{O}_p \text{ module}\}$. A singular distribution θ is called *regular* if $S(\theta) = \emptyset$. On $M \setminus S(\theta)$ there exists a unique analytic subbundle L of $TM|_{M\setminus S(\theta)}$ such that θ is the sheaf of analytic sections of L. We assume that the dimension of the $\mathbb{K}$ vector space L_p is the same for all points p in $M \setminus S$ (this always holds if M is connected). This dimension is called the *leaf dimension* of θ. In what follows, we will mainly consider singular foliations with leaf dimension 1, which are called *singular* 1*-dimensional distributions*.
  A blowing-up $\sigma : (\widetilde{M}, \widetilde{E}) \to (M, E)$ is *admissible* if the center $\mathcal{C}$ is a closed and regular sub-manifold of M that has simple normal crossings with E. The divisor $\widetilde{E}$ stands for the union of the strict transform of E with the new exceptional divisor F of the blowing-up σ. We note that the divisor $\widetilde{E}$ has simple normal crossings whenever the blowing-up is admissible (see pages 137–138 of [15] for details). Finally, given an admissible blowing-up: