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Skew derivations with algebraic invariants of bounded degree

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ABSTRACT

This paper examines semiprime algebras A with a q-skew σ-derivation δ, where both δ and σ are algebraic. With minor assumptions on the ground field, we show that if the invariants A^δ are algebraic of bounded degree, then A must be finite dimensional. As an application, it is shown that if A is a semiprime Banach algebra and δ is continuous, then whenever A^δ is algebraic, it follows that A is finite dimensional.

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1. Introduction

In [8], it is shown that if a semisimple complex Banach algebra A contains an element whose centralizer is algebraic, then A must be finite dimensional. This is generalized in [11], where it is proved that if a semiprime Banach algebra A, over either $\mathbb{R}$ or $\mathbb{C}$, has an algebraic derivation or automorphism which is continuous and has algebraic invariants, then A is finite dimensional.

In light of the results in [8] and [11], two natural questions arise, one dealing with Banach algebras and the other of a purely algebraic nature.

1. If A is a semiprime Banach algebra with a continuous q-skew σ-derivation δ such that the invariants are algebraic, where both δ and σ are algebraic, must A be finite dimensional?
2. If A is a semiprime algebra with a q-skew σ-derivation δ such that the invariants are algebraic of bounded degree, where both δ and σ are algebraic, must A be finite dimensional or at least algebraic of bounded degree?

The first question is answered in Corollary 8. In the results below, A^δ denotes the invariants of δ in A.

Corollary 8. *Let A be a semiprime Banach algebra over $\mathbb{R}$ or $\mathbb{C}$ with continuous q-skew σ-derivation δ, where both δ and σ are algebraic. Then the following conditions are equivalent:*

1. *A^δ is algebraic,*
2. *$A^\delta/P(A^\delta)$ is algebraic, where $P(A^\delta)$ is the intersection of the prime ideals of A^δ,*
3. *$A^\delta/P(A^\delta)$ is finite dimensional,*
4. *A is finite dimensional.*

The second question is answered in Theorem 6 and Corollary 7.

Theorem 6 and Corollary 7. *Let A be a semiprime K-algebra with a q-skew σ-derivation δ, where both δ and σ are algebraic over K, such that A^δ is algebraic of bounded degree and $\deg_K(A^\delta) + 1 < card(K)$.*

1. *Then A is also algebraic of bounded degree and is a finite direct sum of central simple algebras.*
2. *In addition, if K is perfect, then A is finite dimensional over K.*

In general, algebraic algebras need not be algebraic of bounded degree and semiprime algebraic algebras need not be finite dimensional. However, the situation is very different for Banach algebras. In [5], Grabiner shows that every nil Banach algebra must be