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Jacobian algebras with periodic module category and exponential growth

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ABSTRACT

Recently it was proven by Geiss, Labardini-Fragoso and Schröer in [1] that every Jacobian algebra associated to a triangulation of a closed surface S with a collection of marked points M is tame and Ladkani proved in [2] these algebras are (weakly) symmetric. In this work we show that for these algebras the Auslander–Reiten translation acts 2-periodically on objects. Moreover, we show that excluding only the case of a sphere with 4 (or less) punctures, these algebras are of exponential growth. These results imply that the existing characterization of symmetric tame algebras whose non-projective indecomposable modules are Ω-periodic, has at least a missing class (see [3, Theorem 6.2] or [4]).
As a consequence of the 2-periodical actions of the Auslander–Reiten translation on objects, we have that the Auslander–Reiten quiver of the generalized cluster category $\mathcal{C}_{(S,M)}$ consists only of stable tubes of rank 1 or 2.

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1. Introduction

Let k be an algebraically closed field. A potential W for a quiver Q is, roughly speaking, a linear combination of cyclic paths in the complete path algebra $k\langle\langle Q\rangle\rangle$. The Jacobian algebra $\mathcal{P}(Q,W)$ associated to a quiver with a potential (Q,W) is the quotient of the complete path algebra $k\langle\langle Q\rangle\rangle$ modulo the Jacobian ideal $J(W)$. Here, $J(W)$ is the topological closure of the ideal of $k\langle\langle Q\rangle\rangle$ which is generated by the cyclic derivatives of W with respect to the arrows of Q.

Quivers with potential were introduced in [5] in order to construct additive categorifications of cluster algebras with skew-symmetric exchange matrix. For the just mentioned categorification it is crucial that the potential for Q be non-degenerate, i.e. that it can be mutated along with the quiver arbitrarily, see [5] for more details on quivers with potentials.

In [6] the authors introduced, under some mild hypothesis, for each oriented surface with marked points (S,M) a mutation finite cluster algebra with skew symmetric exchange matrices. More precisely, each triangulation $\mathbb{T}$ of (S,M) by tagged arcs corresponds to a cluster and the corresponding exchange matrix is conveniently coded into a quiver $Q(\mathbb{T})$. Labardini-Fragoso in [7] enhanced this construction by introducing potentials $W(\mathbb{T})$ and showed that these potentials are compatible with mutations. In particular, these potentials are non-degenerate. Ladkani showed that for surfaces with empty boundary and a triangulation $\mathbb{T}$ which has no self-folded triangles the Jacobian algebra $\mathcal{P}(Q(\mathbb{T}),W(\mathbb{T}))$ is symmetric (and in particular finite-dimensional). It follows that for any triangulation $\mathbb{T}'$ of a closed surface (S,M) the Jacobian algebra $\mathcal{P}(Q(\mathbb{T}'),W(\mathbb{T}'))$ is weakly symmetric by [8]. In [1] it is shown by a degeneration argument that these algebras are tame.

Next, following Amiot [9, Sec. 3.4] and Labardini-Fragoso [10, Theorem 4.2] we have a 2-Calabi–Yau triangulated category $\mathcal{C}_{(S,M)}$ together with a family of cluster tilting objects $(T_{\mathbb{T}})_{\mathbb{T}\text{ triangulation of }(S,M)}$, related by Iyama–Yoshino mutations such that $\mathrm{End}_{\mathcal{C}_{(S,M)}}(T(\mathbb{T}))\cong\mathcal{P}(Q(\mathbb{T}),W(\mathbb{T}))^{\mathrm{op}}$ (see for example [11] for details of n-Calabi–Yau categories).

Theorem. *Let S be a closed oriented surface with a non-empty finite collection M of punctures, excluding only the case of a sphere with 4 (or less) punctures. For an arbitrary tagged triangulation $\mathbb{T}$, the Jacobian algebra $\mathcal{P}(Q(\mathbb{T}),W(\mathbb{T}))$ is symmetric, tame, its stable Auslander–Reiten quiver consists only of stable tubes of rank 1 or 2 and it is an algebra of exponential growth.*

Recall that for symmetric algebras there is a relation between the Auslander–Reiten translation τ and the Heller translate or syzygy Ω, namely $\tau\cong\Omega^2$ (see [12, Section 2.5]), then if $\Lambda_{\mathbb{T}}=\mathcal{P}(Q(\mathbb{T}),W(\mathbb{T}))$ is a Jacobian algebra as in the main Theorem, we have that the non-projective indecomposable $\Lambda_{\mathbb{T}}$-modules are Ω-periodic. Related to symmetric tame algebras, Erdmann and Skowroński have announced the following statement (see