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Functorial constructions for non-associative algebras with applications to quasi-bialgebras [☆]



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ABSTRACT

The aim of this paper is to establish a contravariant adjunction between the category of quasi-bialgebras and a suitable full subcategory of dual quasi-bialgebras, adapting the notion of finite dual to this framework. Various functorial constructions involving non-associative algebras and non-coassociative coalgebras are then carried out. Several examples illustrating our methods are expounded as well.

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1. Introduction

Algebras and coalgebras are dual notions, in the sense that the latter ones are obtained from the first ones by reversing the structure arrows, that is using the opposite base category. Furthermore, over vector spaces there is a contravariant adjunction (or duality) between the category of algebras and that of coalgebras, whose functors are described as follows. In one direction, with each coalgebra one associates, in a functorial way, its convolution algebra. In the other direction, with each algebra one associates, in a similar way, its topological dual (i.e. finite dual) coalgebra. This adjunction descends in fact to the category of bialgebras (and in particular to Hopf algebras), and also establishes a contravariant adjunction between the category of Lie algebras and the category of Lie coalgebras. All these adjunctions and other ones are captured by the following diagram

$$\begin{array}{ccc}
 \text{Alg}_{\mathbb{k}} & \begin{array}{c} \xleftarrow{(-)^{\circ}} \\ \xleftarrow{(-)^{*}} \end{array} & \text{Coalg}_{\mathbb{k}} \\
 \uparrow & & \uparrow \\
 \text{Bialg}_{\mathbb{k}} & \begin{array}{c} \xleftarrow{(-)^{\circ}} \\ \xleftarrow{(-)^{\circ}} \end{array} & \text{Bialg}_{\mathbb{k}} \\
 \begin{array}{c} \mathcal{U} \updownarrow \mathcal{P} \end{array} & \begin{array}{c} \xleftarrow{(-)^{\circ}} \\ \xleftarrow{(-)^{\bullet}} \end{array} & \begin{array}{c} \mathcal{P}^c \updownarrow \mathcal{U}^c \end{array} \\
 \text{Lie}_{\mathbb{k}} & \begin{array}{c} \xleftarrow{(-)^{\bullet}} \\ \xleftarrow{(-)^{*}} \end{array} & \text{LieCo}_{\mathbb{k}}
 \end{array} \tag{1}$$

where $\mathbb{k}$ denotes the base field, and the notations for the involved categories as well as the ones used in the sequel, are summarized in Table 1 below.

It is noteworthy to mention that in diagram (1), the functor $\mathcal{P} : \text{Bialg}_{\mathbb{k}} \rightarrow \text{Lie}_{\mathbb{k}}$ associates with each bialgebra its Lie algebra of primitive elements, and its left adjoint $\mathcal{U} : \text{Lie}_{\mathbb{k}} \rightarrow \text{Bialg}_{\mathbb{k}}$ is the universal enveloping algebra functor. Note that in characteristic zero there is a natural isomorphism $\mathcal{P}\mathcal{U} \cong \text{id}_{\text{Lie}_{\mathbb{k}}}$, see [16, Theorem 5.18]. The functor $\mathcal{P}^c$ is the one given by the vector space of indecomposables, see e.g. [15, Definition 1.9] where this functor is denoted by Q , and $\mathcal{U}^c$ is a functor defined by Michaelis. The adjunction $\mathcal{P}^c \dashv \mathcal{U}^c$ is established in [15, Theorem 3.11] for Hopf algebras instead of bialgebras (the same proof can be adapted to our case, as the antipode is not used therein). The bottom contravariant adjunction is established in [15, Theorem 3.7]. For the top horizontal adjunction see e.g. [19, Theorem 6.0.5]. Concerning the middle horizontal adjunction, the finite dual yields an endofunctor of $\text{Bialg}_{\mathbb{k}}$ in view of [19, Section 6.2]. Moreover, this comes out to be adjoint to itself as in case of Hopf algebras (cf. e.g. [1, page 87]).

Quasi-bialgebras are generalization of ordinary bialgebras, in which the constraint of coassociativity at the coalgebra level is weakened. Dual quasi-bialgebras are in a certain sense a dual notion, which can also be seen as a generalization of bialgebras, by affecting this time the associativity constraint.

The main aim of this paper is to investigate the second horizontal adjunction of diagram (1) in the context of quasi- and dual quasi-bialgebras. Explicitly, we establish

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