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Extended affine root supersystems



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ABSTRACT

The interaction of a Lie algebra $\mathcal{L}$, having a weight space decomposition with respect to a nonzero toral subalgebra, with its corresponding root system forms a powerful tool in the study of the structure of $\mathcal{L}$. This, in particular, suggests a systematic study of the root system apart from its connection with the Lie algebra. Although there have been a lot of researches in this regard on Lie algebra level, such an approach has not been considered on Lie superalgebra level. In this work, we introduce and study extended affine root supersystems which are a generalization of both affine reflection systems and locally finite root supersystems. Extended affine root supersystems appear as the root systems of the super version of extended affine Lie algebras and invariant affine reflection algebras including affine Lie superalgebras. This work provides a framework to study the structure of this kind of Lie superalgebras refereed to as extended affine Lie superalgebras.

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0. Introduction

Lie algebras having a weight space decomposition with respect to a nonzero abelian subalgebra, called a toral subalgebra, form a vast class of Lie algebras. Locally finite split simple Lie algebras [11], extended affine Lie algebras [1], toral type extended affine Lie algebras [2], locally extended affine Lie algebras [10] and invariant affine reflection algebras [12] are examples of such Lie algebras. We can attach to such a Lie algebra a subset of the dual space of its toral subalgebra called the root system. The interaction of such a Lie algebra with its root system offers an approach to study the structure of the Lie algebra via its root system. This in turn provokes a systematic study of the root system apart from its connection with the Lie algebra; see [1], [8], [17] and [12]. Although since 1977, when the concept of Lie superalgebras was introduced [6], there has been a significant number of researches on Lie superalgebras, the mentioned approach on Lie superalgebra level has not been considered in general. The first step towards such an approach is offering an abstract definition of the root system of a Lie superalgebra. In 1996, V. Serganova [15] introduced the notion of generalized root systems as a generalization of finite root systems; see also [4]. The main difference between generalized root systems and finite root systems is the existence of nonzero self-orthogonal roots. Serganova classified irreducible generalized root systems and showed that such root systems are root systems of finite dimensional basic classical simple Lie superalgebras [6] except for type $A(1,1)$. She also gave two alternative definitions for generalized root systems. In a generalized root system for two self-orthogonal roots which are not orthogonal, either their summation or their subtraction (and not both) is again a root while according to the first alternative definition both summation and subtraction of two self-orthogonal roots which are not orthogonal, can be roots; this in particular allows to obtain type $A(1,1)$ as well. In this work, we introduce extended affine root supersystems and systematically study them. Roughly speaking, a spanning set R of a nontrivial vector space over a field $\mathbb{F}$ of characteristic zero, equipped with a symmetric bilinear form, is called an extended affine root supersystem if the root string property is satisfied. R is called a locally finite root supersystem if the form is nondegenerate. Irreducible locally finite root supersystems have been classified in [19]. One also knows from [19] that root string property for a locally finite root supersystem can be replaced by the locally finiteness of the real part. Generalized root systems according to the first alternative definition mentioned above, are nothing but finite locally finite root supersystems defined over the complex numbers. Locally finite root supersystems naturally appear in the theory of locally finite Lie superalgebras; see [13] and [20]. Extended affine root supersystems are extensions of locally finite root supersystems by abelian groups and appear as the root systems of extended affine Lie superalgebras introduced in [20]; in particular the root system of an affine Lie superalgebra [16] is an extended affine root supersystem. The nonzero elements of an extended affine root supersystem are divided into three disjoint parts: One consists of all real roots, i.e., the elements which are not self-orthogonal. The second part is the intersection of the radical of the form with the nonzero elements;

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