Journal of Algebra 443 (2015) 99–125

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

# Finite-dimensional representations of periplectic Lie superalgebras

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A R T I C L E   I N F O

*Article history:*
Received 29 December 2014
Available online 7 August 2015
Communicated by Volodymyr Mazorchuk

*MSC:*
17B67

*Keywords:*
Periplectic Lie superalgebras
Irreducible characters
BGG reciprocity
Tilting modules
Blocks

A B S T R A C T

We study categories of finite-dimensional modules over the periplectic Lie superalgebras and obtain a BGG type reciprocity. In particular, we prove that these categories have only finitely-many blocks. We also compute the characters for irreducible modules over periplectic Lie superalgebras of ranks 2 and 3, and obtain explicit description of the blocks for ranks 2, 3, and 4.


## 1. Introduction

Lie superalgebras and, in particular, their representation theory have enjoyed a recent surge of considerable interest. This is mainly due to recent works that made connection between Lie superalgebras and other branches in classical Lie theory. But it is also due to the significant progresses made recently, especially, in the irreducible character problem for finite-dimensional simple Lie superalgebras since Kac's pioneering work [17] on typical representations of classical Lie superalgebras. Since then the irreducible characters of

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http://dx.doi.org/10.1016/j.jalgebra.2015.06.042
0021-8693/

classical Lie superalgebras for the finite-dimensional modules, and even for modules in the BGG category $\mathcal{O}$, have been worked out in [24,2,14,8,9,1].

While the BGG categories for the other finite-dimensional simple Lie superalgebras are not well understood at present, the irreducible finite-dimensional characters for them have all been worked out and satisfactory answers obtained with the exception of the periplectic Lie superalgebra.

The periplectic Lie superalgebra is the Lie superalgebra preserving an odd non-degenerate symmetric or skew-symmetric bilinear form. It is thus a superanalogue of the orthogonal or symplectic Lie algebra. One of the main reasons why the representation theory of the periplectic Lie superalgebra is still not well understood is that many classical and traditional methods in representation theory do not appear to be applicable. For example, the center of its universal enveloping algebra fails to provide us with information about the blocks in the respective categories.

The present paper attempts to systematically study categories of finite-dimensional representations of the periplectic Lie superalgebras. There are two possible choices of standard modules in these categories, arising from the non-symmetric nature of the set of the odd roots. In particular, we prove that these categories have only finitely-many blocks by using these two standard modules in Theorem 5.11 and Corollary 5.12. One of the main results is a BGG type reciprocity relating projective covers and irreducible objects in the category, which involves both standard modules. We also compute the irreducible character for the periplectic Lie superalgebras of ranks 2 and 3. Furthermore, we study the blocks of the category, and give an explicit description of the blocks in the rank 2, 3, and 4 cases.

In Section 2, besides introducing various notations for the periplectic Lie superalgebras, the categories of modules to be investigated are defined and studied. Odd reflections for the periplectic Lie superalgebra, introduced by Penkov–Serganova, are then recalled. In Section 3 irreducible characters of the periplectic Lie superalgebra of ranks 2 and 3 are computed. In Section 4 the above mentioned BGG type reciprocity is established in Theorem 4.3, while tilting modules are studied in Section 4. Finally, we investigate block structures in Section 5, and obtained explicit description of the blocks of the various categories for small ranks.

## 2. Preliminaries

### *2.1. The Lie superalgebra $\widetilde{p}(n)$*

Throughout, $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}$ is always the periplectic Lie superalgebra $\widetilde{p}(n)$ [18]. Recall that $\mathfrak{g}$ admits a $\mathbb{Z}_2$-compatible $\mathbb{Z}$-gradation $\mathfrak{g} = \mathfrak{g}_{-1} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_{+1}$ where $\mathfrak{g}_0 = \mathfrak{g}_{\bar{0}} = \mathfrak{gl}(n)$ and $\mathfrak{g}_{-1} \cong \wedge^2(\mathbb{C}^{n*})$, $\mathfrak{g}_{+1} \cong S^2(\mathbb{C}^n)$ as $\mathfrak{g}_0$-modules. The standard matrix realization is given by

$$\widetilde{p}(n) = \left\{ \begin{pmatrix} a & b \\ c & -a^t \end{pmatrix},\ \text{where } b \text{ is symmetric and } c \text{ is skew-symmetric} \right\}.$$