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Periodicity of self-injective algebras of polynomial growth [☆]

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ABSTRACT

Let A be an indecomposable representation-infinite tame finite-dimensional algebra of polynomial growth over an algebraically closed field. We prove that A is a periodic algebra with respect to action of the bimodule syzygy operator if and only if A is Morita equivalent to a socle deformation of an orbit algebra $\widehat{B}/G$ where $\widehat{B}$ is the repetitive category of a tubular algebra B and G is an admissible infinite cyclic automorphism group of $\widehat{B}$. The main contribution in the paper is to prove the sufficiency part of this equivalence. It is known that every orbit algebra $\widehat{B}/G$ of a tubular algebra B admits a presentation as an orbit algebra $\mathrm{T}(B)^{(r)}/H$ of an r-fold trivial extension algebra $\mathrm{T}(B)^{(r)}$ of B with respect to free action of a finite cyclic automorphism group H of $\mathrm{T}(B)^{(r)}$. A significant part of the paper is devoted to explicit descriptions of the minimal projective bimodule resolutions of properly chosen ten exceptional self-injective algebras of polynomial growth and showing that all of them are periodic algebras. Then we deduce the periodicity of all algebras socle equivalent to the orbit algebras $\widehat{B}/G = \mathrm{T}(B)^{(r)}/H$ of tubular algebras B from the periodicity of these ten exceptional algebras, using

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Tubular algebra
Polynomial growth

invariance of periodicity for finite Galois coverings and derived equivalences of algebras.

1. Introduction and the main result

Throughout this paper, K will denote a fixed algebraically closed field. By an algebra we mean an associative, finite-dimensional K-algebra with identity, and we denote by $\operatorname{mod} A$ the category of finite-dimensional right A-modules. An algebra A is called *self-injective* if A_A is an injective A-module, or equivalently, the projective modules in $\operatorname{mod} A$ are injective. Any Frobenius algebra, and in particular any symmetric algebra is self-injective. Recall that the algebra A is a *Frobenius algebra* if there is an associative, non-degenerate K-bilinear form $(-,-) : A \times A \to K$, and it is *symmetric* if there is such form which is in addition symmetric. In fact, the basic algebra of an arbitrary self-injective algebra is a Frobenius algebra.

Given a module M in a module category $\operatorname{mod} A$, its *syzygy* is defined to be the kernel $\Omega_A(M)$ of a minimal projective cover of M in $\operatorname{mod} A$.

Usually the module category $\operatorname{mod} A$ has infinitely many isomorphism classes of indecomposable modules and then the syzygy operator Ω_A is a very important tool to construct modules and relate them. For A self-injective, it induces an equivalence of the stable module category $\underline{\operatorname{mod}}\, A$, and its inverse is the shift of a triangulated structure on $\underline{\operatorname{mod}}\, A$. A module M in $\operatorname{mod} A$ is said to be *periodic* if $\Omega_A^n(M) \cong M$ for some $n \geq 1$, and if so the minimal such n is called the Ω_A-period (shortly period) of M. The action of Ω_A on $\operatorname{mod} A$ can effect the algebra structure of A. For example, if all simple modules in $\operatorname{mod} A$ are periodic, then A is a self-injective algebra. Sometimes one can even recover the algebra A and its module category $\operatorname{mod} A$ from the action of Ω_A. For example, the self-injective Nakayama algebras are precisely the algebras A for which Ω_A^2 permutes the isomorphism classes of simple modules in $\operatorname{mod} A$.

An algebra A is defined to be *periodic* if it is periodic viewed as a module over the enveloping algebra $A^e = A^{\mathrm{op}} \otimes_K A$, or equivalently, as an A-A-bimodule. If A is periodic then it is self-injective, and every indecomposable non-projective module in $\operatorname{mod} A$ is periodic. We also note that periodicity of algebras is invariant under derived equivalence. Periodic algebras have interesting connections with group theory, topology, singularity theory, cluster algebras and mathematical physics. The classification of all periodic algebras up to Morita equivalence may be in reach, and is one of the most attractive open problems at present. For tame algebras one can apply techniques and results which have been obtained so far.

The main aim of this paper is to provide a complete classification of all periodic representation-infinite tame algebras of polynomial growth. Recall that an algebra A is *tame* if for each positive integer d, all but finitely many isomorphism classes of