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Singular equivalence of Morita type with level

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ABSTRACT

We generalize the notion of stable equivalence of Morita type and define what is called "singular equivalence of Morita type with level". Such an equivalence induces an equivalence between singular categories. We will also prove that a derived equivalence of standard type induces a singular equivalence of Morita type with level.

1. Introduction

Let k be a commutative ring and let A be a k-algebra. We denote by A-mod the category of all finitely presented left A-modules, and by A-$\underline{\text{mod}}$ the stable module category of A-mod modulo projective modules. The singular category $\mathcal{D}_{\text{sg}}(A)$ of A is defined to be the Verdier quotient of the bounded derived category $\mathcal{D}^b(A)$ of finitely presented modules over A by the full subcategory $\mathcal{K}^b(A\text{-proj})$ consisting of bounded complexes of finitely presented projective A-modules. Two k-algebras A and B are said to be stably equivalent if their stable categories A-$\underline{\text{mod}}$ and B-$\underline{\text{mod}}$ are equivalent as k-categories, and to be singularly equivalent if their singular categories $\mathcal{D}_{\text{sg}}(A)$ and $\mathcal{D}_{\text{sg}}(B)$ are equivalent as triangulated categories.

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A stable equivalence of Morita type introduced by Broué [3] induces an equivalence of stable categories in case k is a field.

Definition 1.1. (See [3].) Let A and B be two k-algebras. We say that $({}_AM_B, {}_BN_A)$ defines a stable equivalence of Morita type if the following conditions are satisfied:

1. M is finitely generated projective as a left A-module and as a right B-module.
2. N is finitely generated projective as a left B-module and as a right A-module.
3. $M \otimes_B N \cong A \oplus P$ for some finitely generated projective A-A-bimodule P, and $N \otimes_A M \cong B \oplus Q$ for some finitely generated projective B-B-bimodule Q.

Very recently analogous to the notion of stable equivalences of Morita type, Xiao-Wu Chen and Long-Gang Sun defined in [5] the concept of singular equivalences of Morita type.

Definition 1.2. (See [5].) Let A and B be two Noetherian k-algebras. We say that $({}_AM_B, {}_BN_A)$ defines a singular equivalence of Morita type if the following conditions are satisfied:

1. M is finitely generated projective as a left A-module and as a right B-module.
2. N is finitely generated projective as a left B-module and as a right A-module.
3. There are bimodule isomorphisms $M \otimes_B N \cong A \oplus P$, where P is finitely presented and of finite projective dimension as an A-A-bimodule and $N \otimes_A M \cong B \oplus Q$, where Q is finitely presented and of finite projective dimension as a B-B-bimodule.

If k is a field and $({}_AM_B, {}_BN_A)$ defines a singular equivalence of Morita type, then

$$M \otimes_B - : \mathcal{D}_{\mathrm{sg}}(B) \to \mathcal{D}_{\mathrm{sg}}(A)$$

is an equivalence of triangulated categories with quasi-inverse

$$N \otimes_A - : \mathcal{D}_{\mathrm{sg}}(A) \to \mathcal{D}_{\mathrm{sg}}(B).$$

In this paper, we generalize Chen and Sun's notion of singular equivalence of Morita type and define a singular equivalence of Morita type with level (cf. Definition 2.1 below). This new concept is very related to derived equivalences, that is, a derived equivalence of standard type induces a singular equivalence of Morita type with level. This generalizes the fact in [17] and [12] that derived equivalences between two self-injective k-algebras induce stable equivalences of Morita type.

Much work has been done to study the invariants under stable equivalence of Morita type (e.g. [13–16]) and singular equivalence of Morita type (e.g. [5,19]). Singular equivalences of Morita type with level also preserve some invariants, for example,