Journal of Algebra 437 (2015) 116–132

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

The Bridgeland's Ringel–Hall algebra associated to an algebra with global dimension at most two ☆

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ARTICLE INFO

Article history:
Received 4 September 2013
Available online 25 May 2015
Communicated by Michel Van den Bergh

MSC:
16G10
16W35

Keywords:
Ringel–Hall algebras
2-periodic complexes
Bridgeland's Ringel–Hall algebras
Tilted algebras
Canonical algebras

ABSTRACT

For any finite dimensional associative algebra with global dimension ≤ 2, we show that there is an embedding from the twisted Ringel–Hall algebra to the Bridgeland's Ringel–Hall algebra. In particular, this result is true for tilted algebras and canonical algebras.

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1. Introduction

For any finite dimensional semi-simple complex Lie algebra, there is a famous theorem saying that its positive roots correspond bijectively to the isomorphism classes of all (finitely dimensional) indecomposable modules of the corresponding hereditary algebra,

☆ Supported partly by the National Natural Science Foundation of China (Grant No. 11126319) and the National 973 Programs (Grant No. 2011CB808003).

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http://dx.doi.org/10.1016/j.jalgebra.2015.04.013
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which was proved by Gabriel [4] for ADE type and then extended by Dlab and Ringel [3] for any type. This result gives a realization of the positive root system of the semi-simple complex Lie algebra from the modules of the hereditary algebra.

To realize the multiplication of the semi-simple complex Lie algebra from the module category of the hereditary algebra, Ringel [17] introduced a Hall algebra from any abelian category with finite morphism spaces, which generalizes Hall's definition from p-groups [8]. He [18] showed that the positive part of the semi-simple complex Lie algebra can be realized via the Hall algebra from the module category of the hereditary algebra. Moreover, he [19] proved that the positive part of the quantized enveloping algebra of the semi-simple complex Lie algebra is isomorphic to the Hall algebra in some twisted form from the module category of the hereditary algebra. Later, Green [6] extended this result to any symmetrizable Kac–Moody Lie algebra case by showing that there is a natural co-multiplication in the Hall algebra of any hereditary algebra.

So a natural question is how one can use the representation theory of hereditary algebras to realize whole but not only the positive part of any symmetrizable Kac–Moody Lie algebra and its quantized enveloping algebra. There have been many attempts to approach such a question. One way was to consider the Drinfeld double of the Hall algebra of any hereditary algebra constructed by Xiao in [24], and in this way he realized the full quantized enveloping algebra. Second way was given by Peng and Xiao in [13] and [14] who used the similar method as Ringel's to construct the Ringel–Hall Lie algebra from any root category, which is the orbit category of all shift square orbits in the derived category of a hereditary algebra. In this way they realized the full symmetrizable Kac–Moody Lie algebra. Third way was introduced by Toën in [23] who constructed the derived Ringel–Hall algebra from the derived category. Recently, Bridgeland [1] gave a more clever way to consider directly the $\mathbb{Z}_2$-graded complexes (or 2-cycle complexes as called in [13], or 2-periodic complexes as in some literature) of projective modules of an algebra with finite global dimension to construct the Ringel–Hall algebra. He showed that the full quantized enveloping algebra can be also realized by his Ringel–Hall algebra. Later, Yanagida [26] proved that in hereditary algebra case the Bridgeland's Ringel–Hall algebra is isomorphic to the Drinfeld double of the Hall algebra. Inspired by work of Bridgeland, Grosky recently in [7] constructed the semi-derived Ringel–Hall algebras from complex categories or $\mathbb{Z}_2$-graded complex categories. In each way above there also were some further researches, see [11,21,22,15,2,9,12,20,25], and so on.

In this paper we only consider Bridgeland's Ringel–Hall algebras. For any hereditary algebra, Bridgeland [1] has proven that the twisted Ringel–Hall algebra can be naturally embedded in the Bridgeland's Ringel–Hall algebra. We extend this result to the case of any algebra with global dimension ≤ 2. Some special interesting consequences should be for tilted algebras and canonical algebras. Such two kinds of algebras are very important and they both have global dimension ≤ 2.

Notation. We fix a field $k = \mathbb{F}_q$ with q elements, and set $t = \sqrt{q}$. We write $\mathrm{Iso}(\mathcal{A})$ for the set of isomorphism classes of a small category $\mathcal{A}$ and write $[A]$ for the iso-

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