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On strongly F-regular inversion of adjunction

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In this article we give two independent proofs of the positive characteristic analog of the log terminal inversion of adjunction. We show that for a pair $(X, S + B)$ in characteristic $p > 0$, if (S^n, B_{S^n}) is strongly F-regular, then S is normal and $(X, S+B)$ is purely F-regular near S. We also answer affirmatively an open question about the equality of F-Different and Different.

Contents

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1. Introduction

In characteristic 0 it is well known that if $(X, S + B)$ is a pair where $\lfloor S + B \rfloor = S$ is irreducible and reduced, then $(X, S + B)$ is plt near S if and only if (S^n, B_{S^n}) is klt, where $S^n \to S$ is the normalization of S and $K_{S^n} + B_{S^n} = (K_X + S + B)|_{S^n}$ is defined by adjunction. The proof follows from the resolution of singularities and the relative Kawamata–Viehweg vanishing theorem. In characteristic $p > 0$ and in the higher dimension (dim > 3) the existence of the resolution of singularities is not known and the Kawamata–Viehweg vanishing theorem is known to fail, so we cannot expect a similar proof here. In this article we give two independent proofs of the characteristic $p > 0$ analog of the *'Log terminal inversion of adjunction'* mentioned above. We prove the following theorem.

Theorem A *(Theorem 4.1, Corollary 5.4). Let $(X, S+B)$ be a pair where X is a normal variety, $S+B \geq 0$ is a $\mathbb{Q}$-divisor, K_X+S+B is $\mathbb{Q}$-Cartier and $S = \lfloor S+B \rfloor$ is reduced and irreducible. Let $\nu : S^n \to S$ be the normalization and write $(K_X+S+B)|_{S^n} = K_{S^n}+B_{S^n}$. If (S^n, B_{S^n}) is strongly F-regular then S is normal, furthermore S is a unique center of sharp F-purity of $(X, S + B)$ in a neighborhood of S and $(X, S + B)$ is purely F-regular near S.*

The first proof (Theorem 4.1) is a geometric proof based on characteristic 0 type of techniques and the second one (Corollary 5.4) is by characteristic $p > 0$ techniques.

We also answer affirmatively an open question about the equality of the F-Different and the Different asked by Schwede in [25]. Our second proof (Corollary 5.4) of the inversion of adjunction is an application of the equality of these two Differents combined with various known but non-trivial results in characteristic $p > 0$ (see [25,4] and [30]). Our proof of this equality also closes the gap in Takagi's proof of the equality of restriction of certain generalizations of test ideal sheaves (see [30, Theorem 4.4]), where it is assumed that these two Differents coincide.

We prove the following theorem.

Theorem B *(Theorem 5.3). Let $(X, S + \Delta \geq 0)$ be a pair, where X is a F-finite normal excellent scheme of pure dimension over a field k of characteristic $p > 0$ and $S+\Delta \geq 0$ is a $\mathbb{Q}$-divisor on X such that $(p^e - 1)(K_X + S + \Delta)$ is Cartier for some $e > 0$. Also assume that S is a reduced Weil divisor and $S \wedge \Delta = 0$. Then the F-Different, F-Diff$_{S^n}(\Delta)$ is*