Journal of Algebra 431 (2015) 196–208

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

On partial CAP-subgroups of finite groups

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A R T I C L E I N F O

Article history:
Received 16 November 2014
Available online 20 March 2015
Communicated by E.I. Khukhro

MSC:
20D10
20D20

Keywords:
Finite group
Partial CAP-subgroup
p-soluble group
p-length

A B S T R A C T

Given a chief factor H/K of a finite group G, we say that a subgroup A of G avoids H/K if $H \cap A = K \cap A$; if $HA = KA$, then we say that A covers H/K. If A either covers or avoids the chief factors of some given chief series of G, we say that A is a partial CAP-subgroup of G. Assume that G has a Sylow p-subgroup of order exceeding p^k. If every subgroup of order p^k, where $k \geq 1$, and every subgroup of order 4 (when $p^k = 2$ and the Sylow 2-subgroups are non-abelian) are partial CAP-subgroups of G, then G is p-soluble of p-length at most 1.

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http://dx.doi.org/10.1016/j.jalgebra.2015.01.035

1. Introduction

All groups considered in this paper are finite.

Let H/K be a chief factor and let A be a subgroup of a group G. If $AH = AK$ (equivalently, $(A \cap H)K = H$), then A *covers* H/K, and if $A \cap H = A \cap K$ (equivalently, $(A \cap H)K = K$, then A *avoids* H/K. If A either covers or avoids each chief factor H/K of G, then A is said to have the *cover-avoidance property* in G and is called a *CAP-subgroup* of G.

The cover and avoidance property has been one of the most productive subgroup embedding properties of normal type to clarify the group structure, especially in the soluble universe (for instance [8,10]). In fact, as it is shown in [3, Chapter 4], the cover-avoidance property together with the conjugacy of some of those families characterises solubility. Unfortunately, the cover-avoidance property is not persistent in intermediate subgroups: if H is a CAP-subgroup of the group G and $H \leq K \leq G$, it does not necessarily follow that H is a CAP-subgroup of K (see [7, A, Definition 10.8]). The failure of the cover and avoidance property to hold in intermediate subgroups leads to the following weaker property, which is persistent in subgroups and is also extremely useful in the structural study of the groups.

Definition 1. A subgroup A of a group G is called a *partial CAP-subgroup* of G if there exists a chief series Γ_A of G such that A either covers or avoids each factor of Γ_A (see [9,13] for alternative terminologies).

The most important advances in the analysis of the structural impact of the partial cover-avoidance property have drawn upon the research line consisting in obtaining global information about the group in which the p-subgroups having some given order are partial CAP-subgroups, where p is a prime that we hold fixed. For instance, p-solubility can be characterised by the partial CAP-property of the Sylow p-subgroups [9], and groups with Sylow p-subgroups of order greater than p with maximal subgroups partial CAP-subgroups are p-supersoluble [4]. Furthermore, in [5], groups in which the second maximal subgroups of the Sylow p-subgroups are partial CAP-subgroups are fully described, whereas the partial CAP-property of the minimal subgroups of the Sylow p-subgroups of the generalised Fitting subgroup can be also used to characterise p-solubility [2]. The structure of the groups in which every subgroup of subgroup of order p^2 is a partial CAP-subgroup is described in [1].

This paper features new results about partial CAP-subgroups in the context of the above mentioned ones. In fact we further develop the theory by assuming that subgroups of order $d = p^k$, for some fix integer $k \geq 1$, have the partial cover and avoidance property (and of course, we have to assume that the order of the Sylow p-subgroups exceeds d). In the case $p = 2$, an additional assumption would be required: every cyclic subgroup of order 4 when $d = 2$ and the Sylow 2-subgroups are non-abelian.