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Epimorphisms of pseudo-quadratic polar spaces

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ABSTRACT

We classify the epimorphisms of the buildings $\mathsf{BC}_l(K, K_0, \sigma, L, q)$, $l \geq 2$, of pseudo-quadratic form type. This completes the classification of epimorphisms of irreducible spherical Moufang buildings of rank at least two.

1. Introduction

The aim of this paper is to complete the classification of epimorphisms of irreducible spherical Moufang buildings of rank at least two. For projective planes and spaces defined over a skew field or octonion division algebra K such a classification is given by the work of André [1], Faulkner and Ferrar [3] and Skornjakov [5]. It is shown there that such epimorphisms essentially correspond with the total subrings of K, i.e. subrings $R \subset K$

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such that $K = R \cup (R \setminus \{0\})^{-1}$. In [6] the second author derives some of the structure theory of epimorphisms of irreducible spherical Moufang buildings of rank at least two and uses this to show that when such a building is defined over a field (for a suitable definition), then these epimorphisms are closely related with affine buildings (and their non-discrete generalizations).

In view of these results, the only untreated case is that of the buildings $\mathsf{BC}_l(K, K_0, \sigma, L, q)$ ($l \geq 2$) of pseudo-quadratic form type. We include the buildings $\mathsf{C}_l(K, K_0, \sigma)$ of involutory type in this class, which corresponds to the case $L = 0$. The main difference with the cases handled in [6] is that a total subring of a field always corresponds to a valuation of this field, while this is not true for skew fields in general. As a consequence one can no longer apply the rich theory of affine buildings, meaning that we have to construct the epimorphisms in a different, more ad hoc manner.

The precise statement of our classification can be found in Section 3. We note that in this paper we only consider type-preserving epimorphisms between (thick) buildings.

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2. Polar spaces of pseudo-quadratic form type

In this section we define the polar spaces of interest in this paper. Our approach is based on [7, (16.5)]. Let K be a skew field and σ an involution of K, meaning σ is an anti-automorphism (so $(ab)^\sigma = b^\sigma a^\sigma$) with $\sigma^2 = \mathrm{id}$. Let

$$K_\sigma = \{a + a^\sigma | a \in K\},$$
$$K^\sigma = \{a | a \in K, a^\sigma = a\}.$$

Choose a $K_\sigma \subset K_0 \subset K^\sigma$ containing the element 1, such that for all $t \in K$ we have $t^\sigma K_0 t = K_0$. Such a set is called an *involutory set*. If the characteristic of K is different from 2, then $K_\sigma = K_0 = K^\sigma$. Let L be a right vector space over K. A map $f : L \times L \to K$ is a *skew-hermitian sesquilinear form* on L with respect to σ, if $f(a, b)^\sigma = -f(b, a)$ and $f(at, bu) = t^\sigma f(a, b)u$ for all $a, b \in L$ and $t, u \in K$. A map $q : L \to K$ is a *skew-hermitian pseudo-quadratic form* on L with respect to σ if f on L is a skew-hermitian sesquilinear form with respect to σ, such that the following two conditions are satisfied for all $a, b \in L$ and $t \in K$:

- $q(a + b) \equiv q(a) + q(b) + f(a, b) \bmod K_0$,
- $q(at) \equiv t^\sigma q(a)t \bmod K_0$.

If one moreover has that $q(a) \in K_0$ only if $a = 0$, then we say that q is *anisotropic*. If all of this is satisfied we say that the quintuple (K, K_0, σ, L, q) is an *anisotropic skew-hermitian pseudo-quadratic space*.