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Equivalences between blocks of p-local Mackey algebras

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ABSTRACT

Let G be a finite group and $(K, \mathcal{O}, k)$ be a p-modular system. Let $R = \mathcal{O}$ or k. There is a bijection between the blocks of the group algebra and the blocks of the so-called p-local Mackey algebra $\mu_R^1(G)$. Let b be a block of RG with abelian defect group D. Let b' be its Brauer correspondant in $N_G(D)$. It is conjectured by Broué that the blocks RGb and $RN_G(D)b'$ are derived equivalent. Here we look at equivalences between the corresponding blocks of p-local Mackey algebras. We prove that an analogue of the Broué's conjecture is true for the p-local Mackey algebras in the following cases: for the principal blocks of p-nilpotent groups and for blocks with defect 1.

1. Introduction and preliminaries

Let G be a finite group and $(K, \mathcal{O}, k)$ be a p-modular system. Let $R = \mathcal{O}$ or k. In [23], Thévenaz and Webb proved that there is a bijection $b \mapsto b^\mu$ between the blocks of RG and the primitive central idempotents of $\mu_R^1(G)$, called the blocks of $\mu_R^1(G)$, where $\mu_R^1(G)$ is the so-called p-local Mackey algebra. This bijection preserves the defect groups (which

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are the same as the defect groups of the corresponding block of RG, see Section 5.3 of [5]). So using the Brauer correspondence, we have the following diagram: let b be a block of RG with defect group D and b' be the Brauer correspondent of b in $RN_G(D)$.

$$\begin{array}{ccc} b \in Z(RG) & \longrightarrow & b^{\mu} \in Z(\mu_R^1(G)) \\ \downarrow & & \downarrow \\ b' \in Z(RN_G(D)) & \longrightarrow & b'^{\mu} \in Z(\mu_R^1(N_G(D))). \end{array}$$

If D is abelian, it is conjectured by Broué that the block algebras RGb and $RN_G(D)b'$ are derived equivalent. In [21], the Author proved that if two blocks of group algebras are *splendidly* derived equivalent (in the sense of [18]), then the corresponding blocks of the so-called *cohomological* Mackey algebras are derived equivalent. The cohomological Mackey algebra is a quotient of the p-local Mackey algebra. So, it is a very natural question to ask if the same happens for the corresponding p-local Mackey algebras which contain much more information than their cohomological quotient (see Proposition 3.5 e.g.).

Question 1.1 *(Bouc). Let G be a finite group and b be a block of RG with abelian defect group D. Let b' be its Brauer correspondent in $RN_G(D)$. Is there a derived equivalence $D^b(\mu_R^1(G)b^{\mu}) \cong D^b(\mu_R^1(N_G(D))b'^{\mu})$?*

Unfortunately, the tools which were developed for the cohomological Mackey algebras cannot be used here. Moreover Question 1.1 is much harder than its analogue for the cohomological Mackey algebras and is connected to deep questions of representation of finite groups such as the knowledge of the indecomposable p-permutation modules (see Sections 6 and 7 of [4]). Here, we will not answer this question in general, but we consider it in the following two cases: first for the Mackey algebra of the principal block of p-nilpotent groups, and then for blocks with defect group of order p.

The main result of this paper is the following theorem:

Theorem 1.2. *The answer to Question 1.1 is affirmative in the following cases:*

- *For principal blocks of p-nilpotent groups. Here we do not assume the Sylow p-subgroups to be abelian. Moreover it is a Morita equivalence.*
- *For the blocks with defect group of order p.*

For a non-principal block of a p-nilpotent group, or more generally for a nilpotent block it is not expect for the corresponding block of the Mackey algebra to be Morita equivalent to the Mackey algebra of its defect group (see Example 4.8).

Let b and c be two blocks of group algebras. Then it is likely that the corresponding blocks of p-local Mackey algebras will be equivalent only if there is an equivalence between