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K3 surfaces with an automorphism of order 66, the maximum possible ☆

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ABSTRACT

In each characteristic $p \neq 2, 3$, it was shown in a previous work that the order of an automorphism of a K3 surface is bounded by 66, if finite. Here, it is shown that in each characteristic $p \neq 2, 3$ a K3 surface with a cyclic action of order 66 is unique up to isomorphism. The equation of the unique surface is given explicitly in the tame case ($p \nmid 66$) and in the wild case ($p = 11$).

An automorphism of finite order is called *tame* if its order is prime to the characteristic, and *wild* otherwise. Let X be a K3 surface over an algebraically closed field k of characteristic $p \geq 0$. An automorphism g of X is called *symplectic* if it preserves a non-zero regular 2-form ω_X, and *purely non-symplectic* if no power of g is symplectic except the identity. An automorphism of order a power of p in characteristic $p > 0$ is symplectic, as there is no p-th root of unity.

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In characteristic 0 or $p \neq 2, 3, 11$, Dolgachev (as recorded in Nikulin's paper [10]) gave the first example of a K3 surface with an automorphism of order 66. The K3 surface is birational to the weighted hypersurface in $\mathbf{P}(33, 22, 6, 1)$ defined by

$$D_{66} : x^2 + y^3 + z^{11} + w^{66} = 0. \tag{0.1}$$

The surface D_{66} was described in [4, p. 847]. It has 3 singular points and non-trivial canonical Weil divisor. Its minimal resolution has $K^2 = -3$ and is a three time blow-up of a K3 surface. The affine model $x^2+y^3+z^{11}+1 = 0$ is birational to $y^2+x^3+1-s^{11} = 0$, hence to the elliptic K3 surface in $\mathbf{P}(6, 4, 1, 1)$

$$X_{66} : y^2 + x^3 + t_1^{12} - t_0^{11}t_1 = 0, \tag{0.2}$$

which was later described by Kondō [8]. The surface X_{66} has the automorphism

$$g_{66}(x, y, t) = \left(\zeta_{66}^2 x, \zeta_{66}^3 y, \zeta_{66}^6 t\right) \tag{0.3}$$

of order 66 where $t = t_1/t_0$ and ζ_{66} is a primitive 66th root of unity.

In each characteristic $p \neq 2, 3$, it was shown in [6] that the order of any automorphism of a K3 surface is bounded by 66, if finite. In this paper we characterise K3 surfaces admitting a cyclic action of order 66.

For an automorphism g, tame or wild, of a K3 surface X, we write

$$\mathrm{ord}(g) = m.n$$

if g is of order mn and the homomorphism $\langle g\rangle \to \mathrm{GL}(H^0(X, \Omega_X^2))$ has kernel of order m. A tame automorphism g of order 66 of a K3 surface is purely non-symplectic by [6, Lemmas 4.2 and 4.4], i.e.,

$$\mathrm{ord}(g) = 1.66.$$

Theorem 0.1. *Let k be the field $\mathbb{C}$ of complex numbers or an algebraically closed field of characteristic $p \neq 2, 3, 11$. Let X be a K3 surface defined over k with an automorphism g of order 66. Then*

$$\left(X, \langle g\rangle\right) \cong \left(X_{66}, \langle g_{66}\rangle\right),$$

i.e. there is an isomorphism $f : X \to X_{66}$ such that $f\langle g\rangle f^{-1} = \langle g_{66}\rangle$.

Over $k = \mathbb{C}$, Theorem 0.1 was proved by Kondō [8] under the assumption that g acts trivially on the Picard group of X, then by Machida and Oguiso [9] under the assumption that g is purely non-symplectic. Our proof is characteristic free and does not use the tools in the complex case such as transcendental lattice and the holomorphic Lefschetz formula.