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Large 2-groups of automorphisms of algebraic curves over a field of characteristic 2 ☆

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ABSTRACT

Let S be a 2-subgroup of the $\mathbb{K}$-automorphism group $\mathrm{Aut}(\mathcal{X})$ of an algebraic curve $\mathcal{X}$ of genus $\mathfrak{g}(\mathcal{X})$ defined over an algebraically closed field $\mathbb{K}$ of characteristic 2. It is known that S may be quite large compared to the classical Hurwitz bound $84(\mathfrak{g}(\mathcal{X})-1)$. However, if S fixes no point, then the size of S is smaller than or equal to $4(\mathfrak{g}(\mathcal{X})-1)$. In this paper, we investigate algebraic curves $\mathcal{X}$ with a 2-subgroup S of $\mathrm{Aut}(\mathcal{X})$ having the following properties:

(I) $|S| \geq 8$ and $|S| > 2(\mathfrak{g}(\mathcal{X})-1)$,
(II) S fixes no point on $\mathcal{X}$.

Theorem 1.2 shows that $\mathcal{X}$ is a general curve and that either $|S| = 4(\mathfrak{g}(\mathcal{X})-1)$, or $|S| = 2\mathfrak{g}(\mathcal{X})+2$, or, for every involution $u \in Z(S)$, the quotient curve $\mathcal{X}/\langle u\rangle$ inherits the above properties, that is, it has genus ≥ 2, and its automorphism group $S/\langle u\rangle$ still has properties (I) and (II). In the first two cases, S is completely determined. We also give examples illustrating our results. In particular, for every $\mathfrak{g} = 2^h+1 \geq 9$, we exhibit a (general bielliptic) curve $\mathcal{X}$ of genus $\mathfrak{g}$ whose

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$\mathbb{K}$-automorphism group has a dihedral 2-subgroup S of order $4(\mathfrak{g}-1)$ that fixes no point in $\mathcal{X}$.

1. Introduction

In the present paper, $\mathbb{K}$ is an algebraically closed field of characteristic 2, $\mathcal{X}$ is a (projective, non-singular, geometrically irreducible, algebraic) curve of genus $\mathfrak{g} \geq 2$, $\mathrm{Aut}(\mathcal{X})$ is the $\mathbb{K}$-automorphism group of $\mathcal{X}$, and S is a (non-trivial) subgroup of $\mathrm{Aut}(\mathcal{X})$ whose order is a power of 2.

From previous work of Nakajima [13], the size of S is related to the 2-rank γ of $\mathcal{X}$ which is defined to be the rank of the (elementary abelian) group of the 2-torsion points in the Jacobian variety of $\mathcal{X}$; see [8, Section 6.7]. It is known that $\gamma \leq \mathfrak{g}$. If equality holds then $\mathcal{X}$ is a *general* curve, see [8, Theorem 6.96] and [3]. Nakajima [13, Theorem 1] showed that $|S| \leq 4(\gamma-1)$ for $\gamma > 1$, whereas $|S| \leq 4(\mathfrak{g}-1)$ for $\gamma = 1$. Moreover, [7, Theorem 3.4] states that if $\gamma = 0$, then S has a unique fixed point on $\mathcal{X}$, see also [8, Theorem 11.333]. In the latter case, $|S| \leq 8\mathfrak{g}^2$ by an earlier result of Stichtenoth [17] who also pointed out that this bound is attained by the non-singular model $\mathcal{X}$ of the hyperelliptic curve of genus 2^{k-1} and equation $Y^2 + Y + X^{2^k+1} = 0$.

The above results have given a motivation to investigate the possibilities for $\mathcal{X}$, $\mathfrak{g}$ and S when either $|S|$ is close to $8\mathfrak{g}^2$ (and S fixes a point of $\mathcal{X}$), or $|S|$ is close to $4(\mathfrak{g}-1)$ but S fixes no point of $\mathcal{X}$.

The first possibilities have recently been investigated by Lehr, Matignon and Rocher, see [11,12,15,16]. In [11], it is shown that $|S| \geq 4\mathfrak{g}^2$ only occurs when $\mathcal{X}$ is the non-singular model of the Artin–Schreier curve of equation $Y^q + Y + f(X) = 0$ with $f(X) = XP(X) + cX$ where $P(X)$ is an additive polynomial of $\mathbb{K}[X]$ and q is a power of 2.

To investigate the second possibility the hypotheses below are assumed:

(I) $|S| \geq 8$ and $|S| > 2(\mathfrak{g}-1)$,
(II) S fixes no point on $\mathcal{X}$.

Before stating our results on S we point out the prominent role of central involutions in this context. Let u be a central involution in S, that is an involution $u \in Z(S)$, and consider the associated quotient curve $\bar{\mathcal{X}} = \mathcal{X}/U$ where $U = \langle u \rangle$. The factor group $\bar{S} = S/U$ has order $\frac{1}{2}|S|$ and it is a $\mathbb{K}$-automorphism group of $\bar{\mathcal{X}}$. Also, $\mathfrak{g}-1 \geq 2(\bar{\mathfrak{g}}-1)$ where $\bar{\mathfrak{g}}$ is the genus of $\bar{\mathcal{X}}$. Therefore, either

(A) $\bar{\mathfrak{g}} \leq 1$; or
(B) $\bar{\mathfrak{g}} = 2$ and $|\bar{S}| = 4$; or
(C) $\bar{\mathfrak{g}} \geq 2$, and hypothesis (I) is inherited by $\bar{S}$, viewed as a subgroup of $\mathrm{Aut}(\bar{\mathcal{X}})$, but $\bar{S}$ fixes a point on $\bar{\mathcal{X}}$; or