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Braid groups of imprimitive complex reflection groups

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We obtain new presentations for the imprimitive complex reflection groups of type (de, e, r) and their braid groups $B(de, e, r)$ for $d, r \geq 2$. Diagrams for these presentations are proposed. The presentations have much in common with Coxeter presentations of real reflection groups. They are positive and homogeneous, and give rise to quasi-Garside structures. Diagram automorphisms correspond to group automorphisms. The new presentation shows how the braid group $B(de, e, r)$ is a semidirect product of the braid group of affine type $\widetilde{\mathbf{A}}_{r-1}$ and an infinite cyclic group. Elements of $B(de, e, r)$ are visualised as geometric braids on $r+1$ strings whose first string is pure and whose winding number is a multiple of e. We classify periodic elements, and show that the roots are unique up to conjugacy and that the braid group $B(de, e, r)$ is strongly translation discrete.

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Contents

1. Introduction

1.1. Reflection groups and braid groups

A complex reflection group G on a finite dimensional complex vector space V is a subgroup of $\mathrm{GL}(V)$ generated by complex reflections — nontrivial elements that fix a complex hyperplane in V pointwise. Finite (irreducible) complex reflection groups were classified by Shephard and Todd [48]:

(i) a general infinite family $G(de,e,r)$ for positive integral parameters d,e,r;
(ii) thirty-four exceptions, labelled $G_4, G_5, \ldots, G_{37}$.

For the presentations of the above groups, see [13,14].

Finite complex reflection groups are divided into two main classes: primitive and imprimitive. The general infinite family $G(de,e,r)$ is imprimitive except for $G(1,1,r)$ and $G(de,e,1)$. ($G(1,1,r)$ is the symmetric group of degree r and $G(de,e,1)$ is the cyclic group of order d.) The exceptional groups $G_4, G_5, \ldots, G_{37}$ are primitive.

The complex reflection group of type (de,e,r) is defined as

$$G(de,e,r) = \left\{ \begin{matrix} r\times r \text{ monomial matrices} \\ (x_{ij}) \text{ over } \{0\}\cup\mu_{de} \end{matrix} \;\Big|\; \prod_{x_{ij}\neq 0} x_{ij}^d = 1 \right\},$$