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Point modules over regular graded skew Clifford algebras

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Results of Vancliff, Van Rompay and Willaert in 1998 [8] prove that point modules over a regular graded Clifford algebra (GCA) are determined by (commutative) quadrics of rank at most two that belong to the quadric system associated to the GCA. In 2010, in [4], Cassidy and Vancliff generalized the notion of a GCA to that of a graded skew Clifford algebra (GSCA). The results in this article show that the results of [8] may be extended, with suitable modification, to GSCAs. In particular, using the notion of μ-rank introduced recently by the authors in [9], the point modules over a regular GSCA are determined by (noncommutative) quadrics of μ-rank at most two that belong to the noncommutative quadric system associated to the GSCA.

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Introduction

The notion of a graded skew Clifford algebra (GSCA) was introduced in [4], and it is an algebra that may be viewed as a quantized analog of a graded Clifford algebra (GCA). In [4], it was shown that many of the results that hold for GCAs have analogous counterparts in the context of GSCAs. In particular, homological and algebraic properties of a GSCA are determined by properties of a certain quadric system associated to the GSCA. The importance of GSCAs was highlighted in [6], where they were shown to play a critical role in the classification of the quadratic Artin–Schelter regular (AS-regular) algebras of global dimension three, and, again, in [4], where some families of GSCAs were presented that contain candidates for generic AS-regular algebras of global dimension four (that is, they have twenty distinct point modules and a one-dimensional line scheme); indeed, the only algebras known to date that are candidates for generic AS-regular algebras of global dimension four are GSCAs. Hence, it is thought that GSCAs might play a critical role in the classification of the quadratic AS-regular algebras of global dimension four and greater. The reader is referred to [1–3] for results concerning AS-regular algebras and their associated geometric data, and to [7,8] for results concerning GCAs and their associated geometric data.

Consequently, it is reasonable to attempt to extend the results in [8] concerning point modules over GCAs to point modules over GSCAs. Hence, our main objective in this article is to generalize [8, Theorem 1.7]. That result states, in part, that if the number, N, of point modules over a regular GCA is finite, then $N = 2r_2 + r_1$, where r_j is the number of elements of rank j that belong to the projectivization of a certain quadric system associated to the GCA (see Theorem 2.10 for the precise statement). We achieve our objective in Theorem 2.12, where the notion of μ-rank (introduced in [9]) is used in place of the traditional notion of rank. However, [8, Theorem 1.7] also states, in part, that if $N < \infty$, then $r_1 \in \{0, 1\}$. We present examples in the last section that demonstrate that this part of [8, Theorem 1.7] appears not to have an obvious counterpart in the setting of GSCAs.

Although the flow of this article follows that of [8, §1], many of our results require methods of proof that differ substantially from those used in [8, §1], since the proofs in [8] make use of standard results concerning symmetric matrices and the general linear group. This article consists of two sections: in Section 1, notation and terminology are defined, while Section 2 is devoted to proving our main result, which is given in Theorem 2.12.

1. Graded skew Clifford algebras

In this section, we define the notion of a graded skew Clifford algebra from [4], and give the relevant results from [4] needed in Section 2.

Throughout the article, $\Bbbk$ denotes an algebraically closed field such that $\mathrm{char}(\Bbbk) \neq 2$, and $M(n, \Bbbk)$ denotes the vector space of $n \times n$ matrices with entries in $\Bbbk$. For a graded $\Bbbk$-algebra B, the span of the homogeneous elements in B of degree i will be denoted B_i,