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Fitting ideals and multiple points of surface parameterizations

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A B S T R A C T

Given a birational parameterization ϕ of an algebraic surface $\mathscr{S} \subset \mathbb{P}^3$, the purpose of this paper is to investigate the sets of points on $\mathscr{S}$ whose pre-image consists of k or more points, counting multiplicities. These points are described explicitly in terms of Fitting ideals of some graded parts of the symmetric algebra associated with the parameterization ϕ. To obtain this description, we show that the degree and dimension of a fiber could be computed by comparing the drop of rank of two explicit (representation) matrices associated with ϕ.

1. Introduction

Parameterized algebraic surfaces are ubiquitous in geometric modeling because they are used to describe the boundary of 3-dimensional shapes. To manipulate them, it is

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very useful to have an implicit representation in addition to their given parametric representation. Indeed, a parametric representation is for instance well adapted for drawing or sampling whereas an implicit representation allows significant improvements in intersection problems that are fundamental operations appearing in geometric processing for visualization, analysis and manufacturing of geometric models. Thus, there exists a rich literature on the change of representation from a parametric to an implicit representation under the classical form of a polynomial implicit equation. Although this problem can always be solved in principle, for instance via Gröbner basis computations, its practical implementation is not enough efficient to be useful in practical applications in geometric modeling for general parameterized surfaces.

In order to overcome this difficulty, alternative implicit representations of parameterized surfaces under the form of a matrix have been considered. The first family of such representations comes from the resultant theory that produces a non-singular matrix whose determinant yields an implicit equation from a given surface parameterization. But the main advantage is also the main drawback of these resultant matrices: since they are universal with respect to the coefficients of the given surface parameterization, they are very easy to build in practice, but they are also very sensitive to the presence of base points. As a consequence, a particular resultant matrix has to be designed for each given particular class of parameterized surfaces. The second family of implicit matrix representations is based on the syzygies of the coordinates of a surface parameterization. Initiated by the geometric modeling community [20], these matrices have been deeply explored in a series of papers (see [6,1,4,8] and the references therein). Compared to the resultant matrices, they are still very easy to build, although not universal, but their sensitivity to the presence of base points is much weaker. However, these matrices are in general singular matrices and the recovering of the classical implicit equation from them is more involved. Therefore, to be useful these matrices have to be seen as implicit representations on their own, without relying on the more classical implicit equation. In this spirit, the use of these singular matrix representations has recently been explored for addressing the curve/surface and surface/surface intersection problems [7,17].

In geometric modeling, it is of vital importance to have a detailed knowledge of the geometry of the object one is working with. In the first step, it is of interest to determine whether a given point on the surface is a singular point or a smooth point, that is, to know whether that point "is being painted twice, or not". In the second step, it is important to give a computationally efficient criterion to answer this question.

To ask the question of how many times is the same point being painted, it should be clear that this question is relevant on the parameterization and not on the surface itself. It is for this reason that (among computational difficulties) when studying directly the implicit equation of the surface, this essential information is lost, while the direct treatment of searching multiple pre-images often involves questions of existence that are computationally difficult to implement. In this paper, we show that this information is all contained in the above-mentioned matrix representations of the parameterization.