Journal of Algebra 424 (2015) 522–539

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Normality and K_1-stability of Roy's elementary orthogonal group

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A R T I C L E I N F O

Article history:
Received 9 January 2014
Available online 16 October 2014
Communicated by Luchezar L. Avramov

MSC:
19G05
19B10
19G99
19B99
20H25

Keywords:
Quadratic modules
Dickson–Siegel–Eichler–Roy transformations
Normality
Stability

A B S T R A C T

In this paper, we prove the normality of the elementary orthogonal group (Dickson–Siegel–Eichler–Roy or DSER group) over a commutative ring which was introduced by A. Roy in [14] under some conditions on the hyperbolic rank. We also establish stability theorems for K_1 of Roy's group under different stable range conditions. We obtain a decomposition theorem for the elementary orthogonal group which is used to deduce the stability theorem.

1. Introduction

In 1960s, H. Bass initiated the study of the normal subgroup structure of linear groups. He introduced a new notion of dimension of rings, called stable rank, and proved that the principal structure theorems hold for groups whose degrees are large with respect to

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http://dx.doi.org/10.1016/j.jalgebra.2014.07.036

the stable rank. Later, J.S. Wilson, I.Z. Golubchik and A.A. Suslin made many other important contributions in this direction. In 1977, A.A. Suslin proved that over any commutative ring A, the group $\mathrm{E}_n(A)$ is always normal in $\mathrm{GL}_n(A)$ when $n \geq 3$.

The normal subgroup structure of symplectic and classical unitary groups over rings were studied by V.I. Kopeĭko in [9], G. Taddei in [17] and by Suslin and Kopeĭko in [15]. Similar results were obtained for general quadratic groups by A. Bak, V. Petrov, and G. Tang in [6], for general Hermitian groups by G. Tang in [18] and A. Bak and G. Tang in [5], and for odd unitary groups by V. Petrov in [11] and W. Yu in [20].

The stability problem for K_1 of quadratic forms was studied in 1960s and in early 1970s by H. Bass, A. Bak, A. Roy, M. Kolster and L.N. Vaserstein. The stability theorems relate unitary groups and their elementary subgroups in different ranges. The stability results for quadratic K_1 are due to A. Bak, V. Petrov and G. Tang (see [6]), and for Hermitian K_1 are due to A. Bak and G. Tang (see [5]). Recently, in [20], W. Yu proved the K_1-stability for odd unitary groups which were introduced by V. Petrov. Stronger results for spaces over semilocal rings are due to A. Roy and M. Knebusch for quadratic spaces (see [14,8]) and H. Reiter for Hermitian spaces (see [13]). In [16], S. Sinchuk proved injective stability for unitary K_1 under stable range condition.

In this paper, we study the elementary orthogonal transformations introduced by A. Roy in [14] for quadratic spaces with a hyperbolic summand over a commutative ring. These transformations over fields are known as *Siegel* transformations or *Eichler* transformations in the literature. We call the group generated by these transformations the Dickson–Siegel–Eichler–Roy (DSER) orthogonal group. We prove normality results and the stability results under Λ-stable range condition. We also prove the injective stability for K_1 of the orthogonal group under stable range condition. A useful tool in the proof will be a decomposition theorem for the elementary subgroup that we will establish in Section 6. For proving stability, we adapt the method used in [6,5,16]. We use some commutator relations which are proved in [1].

In [11], V. Petrov introduced a new classical-type group called odd unitary group over odd form rings which generalizes and unifies all known classical groups. The elementary orthogonal group considered in this article coincides with Petrov's odd hyperbolic unitary group over commutative rings when the module is free. This is proved in [2]. Comparison of this group with isotropic reductive groups defined in [12] reveals that Roy's group is contained in the elementary group considered in [12, Section 7, Example 2]. This comparison is included in a forthcoming article [4].

2. Preliminaries

Let A be a commutative ring with identity in which 2 is invertible. Let (Q, q) be a quadratic space and P be a finitely generated projective module. Consider the orthogonal sum $Q \perp H(P)$ of quadratic spaces Q and $H(P)$, where $H(P)$ denote the hyperbolic space. Here, Q is equipped with a non-singular quadratic form q and $H(P) = P \oplus P^*$ has the natural quadratic form given by $p((x, f)) = f(x)$ for $x \in P$, $f \in P^*$. The quadratic space