



ELSEVIER

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra



Castelnuovo–Mumford regularity, postulation numbers and relation types



Markus Brodmann^a, Cao Huy Linh^{b,1}

^a *Institute of Mathematics, University of Zurich, Winterthurerstrasse 190, CH-8057 Zurich, Switzerland*

^b *Department of Mathematics, College of Education, Hue University, 34 Le Loi, Hue, Vietnam*

ARTICLE INFO

Article history:

Received 26 July 2013

Available online 25 August 2014

Communicated by Steven Dale

Cutkosky

Dedicated to Prof. Ngo Viet Trung on the occasion of his 60th birthday

MSC:

primary 13D45, 13D07

secondary 14B15

Keywords:

Associated graded rings

Castelnuovo–Mumford regularity

Postulation number

Relation type

Uniform bound

ABSTRACT

We establish a bound for the Castelnuovo–Mumford regularity of the associated graded ring $G_I(A)$ of an \mathfrak{m} -primary ideal I of a local Noetherian ring (A, \mathfrak{m}) in terms of the dimension of A , the relation type and the number of generators of I . As a consequence, we obtain that the existence of uniform bounds for the regularity of the associated graded ring, and the relation type of parameter ideals in A , are equivalent conditions. In addition, we establish an equation for the postulation number and the Castelnuovo–Mumford regularity of the associated graded ring $G_{\mathfrak{q}}(A)$ of a parameter ideal \mathfrak{q} , which holds under certain conditions on the depths of the occurring rings. We also show, that the regularity of the ring $G_{\mathfrak{q}}(A)$ is bounded in terms of the dimension of A , the length of A/\mathfrak{q} and the postulation number of $G_{\mathfrak{q}}(A)$.

© 2014 Elsevier Inc. All rights reserved.

E-mail addresses: brodmann@math.uzh.ch (M. Brodmann), huylinh2002@yahoo.com (C.H. Linh).

¹ Cao Huy Linh is supported by the NAFOSTED of Vietnam under grant number 101.01-2012.18.

1. Introduction

Let (A, \mathfrak{m}) be a Noetherian local ring and let $I \subset A$ be an \mathfrak{m} -primary ideal. We denote by $G_I(A) := \bigoplus_{n \geq 0} I^n/I^{n+1}$ the associated graded ring of A with respect to I . The Castelnuovo–Mumford regularity $\text{reg}(G_I(A))$ of the associated graded ring $G_I(A)$ provides upper bounds for some other important invariants of I such as the reduction number $r(I)$, the relation type $\text{reltype}(I)$, and the postulation number $n(I)$ of I . More precisely, it holds that

$$\max\{n(I), r(I), \text{reltype}(I) - 1\} \leq \text{reg}(G_I(A)).$$

A most important issue is the question, whether the previously mentioned invariants have uniform upper bounds as I runs through all parameter ideals of A . More precisely, if there exists a number N such that $\text{reg}(G_{\mathfrak{q}}(A)) \leq N$ (or $\text{reltype}(\mathfrak{q}) \leq N$, or $n(\mathfrak{q}) \leq N$) for all parameter ideals \mathfrak{q} of A , then we say that A has a uniform bound for the regularity of the associated graded ring (respectively the relation type, or the postulation number) of parameter ideals. The search for such uniform bounds was initiated by Huneke’s Relation Type Conjecture (see [1, Question 1.1]), that is the question:

Question 0. Does there exist a uniform bound for the relation type of parameter ideals of a complete equidimensional Noetherian local ring?

The Relation Type Conjecture has attracted a lot of attention. So, for example Lai [6] showed that this conjecture holds for generalized Cohen–Macaulay rings under the assumption that the residue field is infinite. Later, Wang [13] proved that the conjecture holds for generalized Cohen–Macaulay rings without any restriction on the residue field. In [14], Wang showed that the above conjecture holds for all two-dimensional Noetherian local rings. In [1], Aberbach, Ghezzi and Ha provided an example showing that the conjecture need not hold for local rings with two-dimensional non-Cohen–Macaulay locus. Moreover, they showed that the conjecture holds in formally unmixed Noetherian local rings whose formal homology-multiplier ideal is a prime of dimension one. This answers Huneke’s Relation Type Conjecture affirmatively in the formally unmixed case and in a situation which implies that the non-Cohen–Macaulay locus is of dimension one. It is still an open problem, whether Huneke’s conjecture holds for arbitrary formally unmixed Noetherian local rings with one-dimensional non-Cohen–Macaulay locus.

In [8], Linh and Trung established a uniform bound for the regularity of parameter ideals in generalized Cohen–Macaulay rings. Clearly, by the previously mentioned estimate, a uniform bound for the regularity of associated graded rings of parameter ideals implies a uniform bound for the relation type and also for the postulation number of such ideals. In relation with this observation, Ngo Viet Trung raised the following two questions:

Download English Version:

<https://daneshyari.com/en/article/4584630>

Download Persian Version:

<https://daneshyari.com/article/4584630>

[Daneshyari.com](https://daneshyari.com)